Detection of local faults by changes in modal parameters

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Abstract

Detection of local faults is a subject for ongoing research and importance to mechanical systems and structures. Two methods of assessing the changes in vibration modal parameters of a beam, to detect local faults are compared.

The introduction of simulated local fault in the form of an open crack is introduced into a mild steel beam and detection of the local fault is obtained.

The first method used to assess the change in modal parameters is the assessment of the changes in Young’s Modulus in a modal model updating process where the parameters of a finite element model is allowed to vary such that the response, of mode shapes and natural frequency, differences between the finite element model and the experimentally measured responses are minimized.

The second method assesses the change in modal curvature of the experimentally measured mode shapes with the greatest change in curvature at the point of the simulated fault.

Both methods of assessment show clearly the presence and location of the simulated local fault through the changes in the measured modal parameters and their appropriate processing for each method used.

Severity of the simulated crack is also able to be detected with greater changes in parameters with an increased simulated crack depth.

Comparisons of both methods ability to detect the simulated crack in the presence of noise, amount of measurements needed and data manipulation required to assess the results, resulting in the modal model updating process with the use of only the natural frequencies as responses shown to be the best method for fault detection in most instances.
Signed Statement

I declare that all the work and ideas presented within are the original work of the author. All reasonable effort has been taken in ensuring that, to the best of my knowledge, any work presented within by others has been appropriately sighted and referenced.

Signed ____________________________ Date ____________________________

Gareth Forbes
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The construction of this thesis and the results and ideas that are presented within was conducted with a great deal of help, support and guidance from laboratory staff and my thesis supervisor.

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<td>I</td>
<td>Moment of Inertia</td>
</tr>
<tr>
<td>κ</td>
<td>Curvature</td>
</tr>
<tr>
<td>ν</td>
<td>Curvature</td>
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<td>$V(x,t)$</td>
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1 Introduction

The basis on which this document is written and the formulation of the structure is presented within. Project objectives are firstly explained in detail before an overview of the project in full is considered. By the end of this section a clear understanding of the project, its entire overview as well as objectives which need to be attained for the successful evaluation of the project and its performance, and the methods proposed ability to detect a local fault in a structure.

Chapter layout is shown so that the most effective way of moving through the document can be achieved and all presented arguments can be understood with the most productive use of time.

1.1 Project Objectives

The very essence of this project is to measure the changes in modal parameters of a system such that a local change within the structure can be found by the subsequent change in the structures modal parameters. The project set out to evaluate two methods of interpreting the change of these modal parameters and methods to represent and display their changes. Upon investigation of the two basis methods that were first proposed to evaluate the change in modal parameters of the system, some other methods were in fact discovered and also evaluated. A comparison of the methods is then to be undertaken so that a choice between both presented methods ability to detect and locate a fault within the structure can be made.

1.2 Project Overview

As was previously stated the objectives of this project is the detection of a local fault within a beam by changes in the modal parameters. The local fault is in fact
a simulated open crack, meaning that upon deflection the crack does not touch across the crack surface. Two cracks were introduced at different locations with varying depth and width for each. The first crack is located just off centre and measurements were taken with a crack depth at one quarter way through the cross section and then again at half way through the cross section. The second crack was introduced 20% along the length of the beam at a depth of half way through the cross section and with a width around four times wider than the initial crack.

The beam was set up with free-free boundary conditions, and vibration measurements were taken over the 1 metre long beam across a grid of 21 measurements along the length and 3 deep across the width, given 63 measurement points in total. The excitation was applied at one corner of the beam, with a broadband excitation vibration applied through a dynamic shaker. Signal processing techniques were used to obtain the transfer of the vibration signal through the beam such that systems response to a vibration input can be found through the modal parameters of natural frequency, damping and mode shapes. The signal process used to obtain these modal parameters was the Frequency Response Function, FRF, which is in general the ratio of input to output at all the measurement points.

Analysis of the change in the modal parameters is then undertaken by two specific techniques to detect and locate the simulated crack in the structure. The modal parameters that were used in the techniques are natural frequency, mode shapes and modal curvatures.

The first method used to locate the crack within the beam was a sensitivity based modal model updating technique. This techniques’ uses both the classical methods of vibration analysis measurements, and the more modern analysis tool of Finite Element Analysis, FEA. The modal model updating technique uses a finite
element model of the modal parameters, and then offers the ability to change the structural parameters of the FEA model such that the modal parameters found experimentally match the analytical modal parameters. The modal parameters of the analytical FEA model are forced to match the experimental modal parameters in a least squares manner by iterative estimation of the structural parameters of the FEA model which are permitted to vary. This method is proposed to give the ability to match the modal parameters in a way so that the noise within the experimental measurements are not forced into the response of the FEA model [1]. The introduction of the simulated crack should cause a local change in the stiffness of the structure which can be simulated by allowing the stiffness of the elements of the finite element model to vary. Therefore the greatest change in stiffness at the location of the crack should be found such that both the relative change in parameters and there location should give the severity and position of a fault within the structure.

The second method used to assess the change in modal parameters due to the introduction of the simulated crack so that the location and severity of this crack can be found uses only the modal parameter of mode shape curvature. Analysis of the mode shapes curvature is conducted solely from the experimentally measured mode shapes, however some FEA simulation of the beam and the local fault were undertaken to show the fundamental ability of the method.

As the introduction of a crack into a structure means a reduced area over which forces can be transmitted, then it would cause a local change in stiffness at its position. A method therefore that directly exploits the change in local stiffness would be an ideal method of crack detection. The second method of detection in fact uses this in analysis of mode shapes curvature. From the bending moment curvature equation, $M_{moment} = EI\kappa$, where $\kappa$ is the curvature and the other symbols take there...
usual meaning. Observation of this equation shows that any change in area in which the crack causes in the structure this will cause a local discrete change in curvature. This discrete change in curvature is to be both a method of detection of the presence of the crack and also its location along the beams length an indicator of the simulated faults location.

Upon analysis of the two above methods some other forms of fault detection were also explored and will be covered within. Some of these other forms used to locate the simulated crack are, MAC, COMAC and curvature Kurtosis analysis.

### 1.3 Goals for Evaluation

A method for evaluation of the performance of all the methods used to locate the simulated crack is a principle idea which needs to be at least qualitatively, if not quantitatively outlined before the reader should continue.

It is trivial that the presence of the simulated crack needs to be able to distinguished from the measurements or simulation results in the case of FEA for any one of the methods to have any sort of validation. However a higher degree of performance is to be set by showing that the method in fact shows the ability to allow a user with knowledge of the system and methods being used to detect the presence and location of a local fault without prior knowledge of its presence or location.

To build upon this ability of crack detection without a prior knowledge of its existence, the methods shall also be evaluated on their ability to withstand measurement noise within the measurements and detection within this measurement noise. A minimum amount of fault severity that each method can detect will be postulated and used to critic each methods performance and validity.

Finally the amount of measurements, data manipulation, user effort and computation time will be used as a criterion for performance.
1.4 Chapter Layout/Structure

The layout and structure of each chapter will be held constant throughout the thesis. Each chapter will have a short introduction just after the chapter title with then the series of sub headings with the details of the chapter then worked through. The flow of the chapters through the thesis is governed by the way in which the learning process takes place and need to know the knowledge of the preceding chapters to work through the next and build on. The flow chart below in Figure 1.1 shows the basic layout of chapters and data flow process.

The chapters are arranged in the order shown in Figure 1.1, with each block of knowledge needed to be known before you can move down the line to the next block of knowledge. Where there is two paths from one data block, you can move down either line without first acquiring the knowledge of the other path. This means that although all the chapters should be read in order, it is able to read the chapters out of order through some parts of the thesis without loss of continuity in the knowledge path.
Figure 1.1 (Knowledge, chapter and data transfer flow chart)
2 Literature Review

A review of current theories and developments throughout history on the topic of fault detection in structures and related work is to be presented within this chapter. Revision of previous work helps introduce the reader to the topic and its relevance within the engineering field, as well as giving backing to the ideas which are explored within the thesis itself. Development of ideas for future work on the detection of local faults are also able to be found within this chapter from the comparison of other forms of detection to exploration of new methods.

Over fifteen references have been quoted and can be divided into three main categories of ideas are reviewed within, the first being the introduction of ideas for fault detection but not directly related to the methods proposed within the thesis itself. The other two categories can be divided into to either being examples or closely related ideas of the Modal Model Updating fault detection method, model based approach, or the change in Modal Curvature method, direct response based approach.

2.1 Related work and developed concepts

Objective analysis and quantitative methods of structural condition assessment using vibrational parameters has been the subject of much research within engineering studies for the past two decades. A large number of specific methods have surfaced and have shown reasonable success in certain situations for condition assessment such that damage detection can be localized and quantified, however no one method has been able to objectively move to the forefront as the universally accepted method. Specific methods based on the measurement of a structures vibrational parameters and the inference of the presence and location of damage from the values measured and obtained has probably been thought of as the most promising area of research in
structural damage detection. The accessibility of adequate measurement techniques has been a large driving force behind these vibration based techniques. Inspection of structural components for damage is vital for the ongoing maintenance and use of any structure or mechanical system. The location of damage may occur at sections of any structure in a position where visual inspection can not be undertaken, or would not detect damage. Visual methods of damage detection are also not often able to give a quantifiable measure of the damage [18], so a more rigorous scientific based method is also needed, with vibrational measurement methods being ideal in the above situations.

Structural damage detection by vibrational measurements, are all based upon some form of analysis of the change in the derived modal parameters from the experimental measurements. Modal parameters often used for analysis are the natural frequencies, damping ratios, mode shapes and modal curvatures, any form of modal parameter can be used for assessing their changes due to structural defects but the preceding list are the most widely researched and used parameters.

Initial research into structural damage detection by vibrational measurements methods used the measured natural frequencies of the structure, and comparison to undamaged structures could show a change in frequency which would indicate a change in the structure, and possible damage. This initial crude method of only assessing changes in natural frequency will only give a measure of global fault detection and will fail to give any spatial information to the damage within the structure. Analysis of the stress distribution between different mode shapes will show that the distribution is infact different of each mode and non-uniform across the structure [18][20]. This means that the location of the crack will effect each mode differently, and as such the ratio of undamaged to damaged eigenfrequency for a
number of modes can give spatial information to the location of damage. Nahvi et al, exploited this method to both locate the position of damage in a simulated finite element beam as well as its severity [20].

The natural frequencies of a structure are often very insensitive to damage [1][20][17][16][26], this can be intuitively understood by the fact that a continuous beam with free-free boundary conditions and built in end conditions, has the same natural frequencies but very different mode shapes. It is then understandable that the change in mode shape that damage will cause will not show a great deal of change in the natural frequency. Research conducted by Binici [22] used a method of determining the natural frequency and mode shape of a beam after measurement of the initial modal parameters and then with the introduction of simulated cracks and axial forces. The introduction of axial forces change the boundary conditions and is a factor which could change in a lot of structures between measurements due to changed loading conditions, and adequate determination of its effect on mode shapes and natural frequencies is needed to have accurate damage detection.

The insensitivity of the natural frequency to damage in the structure has lead to methods which involve the use of mode shape data in conjunction with natural frequencies or singularly show a measure of damage and its location. Christides et al developed a direct analytical model from the Euler-Bernoulli beam equation for symmetrical open cracks to directly find the mode shapes of a damaged structure [19]. This method uses the simple Euler-Bernoulli equation for the original undamaged cross section to describe the displacement field of the beam in the undamaged location with an exponential decay assumed to the Euler-Bernoulli equation which describes the cross section of the damaged area. Experimental results showed that the developed model was able to accurately predict the natural frequencies, mode shapes
Chapter 2

and crack depth and location. This model is however limited by only being able to model a beam, which is not a complex structure like most real structures needing analysis. The development of this theory to be integrated into a finite element modelling system, using Euler-Bernoulli beam elements to update to the cracked theory Euler-Bernoulli beam elements would be challenging but interesting.

Paradoxically with the greater sensitivity of mode shapes to damage within the structure, mode shapes are also measured with much less certainty than natural frequencies [1] due to measurement noise and technique. This means that further processing of the measured mode shape data is often needed to produce adequate results to base an analysis method on. One method of measuring the change in mode shape directly from the damaged case, with measurement noise was done by Hadjileontiadis et al [15] with measurement of the Kurtosis of the mode shape. The Kurtosis is a higher order statistic which is a measure of the tail shape of a distribution of data. This means that a sharp data distribution would cause a high Kurtosis value. The introduction of a local fault in the form of a simulated crack in a cantilever beam was done be Hadjileontiadis et al, and this causes a local change in mode shape at this position. A moving window in which the Kurtosis is measured is then moved over the mode shape data and the Kurtosis over the length of the beam was shown. The Kurtosis showed a local spike in value at the location of a simulated crack of 30% of cross section with a value approximately twice the height of other spikes in the Kurtosis curve which can be assumed due to measurement noise.

Model based vibrational approaches have received a large amount of attention and use in the last decade, on the back of both the increased use of FEA in all modern engineering simulation, design and diagnostics but also the ability of proprietary software being able to be developed to use solely for use with the updating algorithm.
Mottershead and Friswell have written a comprehensive text on all widely used updating procedures and processes [1]. There exists two basic updating methods, a direct updating method where the orthogonal properties of the eigenvectors to the mass matrix are exploited, and analytical mode shapes and natural frequencies are forced to match those experimentally. This means that all noise within the measurements is replicated but also has the advantage that these direct methods don’t require any iterations. The second model updating method is based upon the sensitivity of a finite element model and the parameter changes are weighted by this sensitivity and parameters are estimated using an appropriate estimation technique such as Bayesian estimation and the response differences are minimised usually in a least squares sense. This means that the analytical responses can be updated to match the experimental responses with the minimum influence by measurement noise [1]. Much research has been undertaken using model updating techniques to estimate damage within structures [18][20][21][24]-[26].

Wu et al shows a good example of a two stage sensitivity based FE model updating technique to estimate structural damage in a complex structure [24]. The first stage involves producing a FEA model of a structure then updating this to the undamaged experimental results first to account for the unknown properties within the structure, such as joint stiffness before a the FEA model can be updated to the damaged model case with structural members removed. The choice of updating parameters was chosen to be Young’s Modulus and natural frequencies used as measurement responses. Natural frequencies are often used as the updating responses due to their ease of measurement and sensitivity to the updating algorithm, mode shapes are often advantageous to be used in the updating responses due to reasons discussed earlier, but a lot of the mode shapes degrees of freedom are insensitive to
the updating algorithm, Jaishi et al used a minimisation of a flexibility residual in an attempt to bring together the mode shapes and natural frequencies in single response. The flexibility is derived from the inverse of the stiffness matrix of the responses so contains information on both the natural frequencies and mode shapes of the structure. It was shown in the study by Jaishi et al that a simulated crack of 20% depth of cross section with an updated parameter value twice the value of non-damaged sections corrupted by a simulated 3% noise input was detected. The experimental results showed the general location of the damage within the beam but, did not show the direct elements with damage.

Some evidence has been shown for the updating model method with the use of the direct use of the Frequency Response Functions, FRF’s, without the need of calculating the mode shapes of the structure. It can be understood that this will lead to less errors in the updating parameters as the direct FRF measurements do not contain assumptions needed by the curve fitting algorithms and also contain data out of measured frequency range [30]. However the FRF’s of the finite element model need then to be synthesised to update against the experimental FRF’s and this means an amount of damping needs to be estimated and will in fact introduce errors just as are introduced with the curve fitting algorithms used in mode shape construction [32]. The density of the FE mesh obviously limits the localisation that can be estimated of the damage by observing the updated parameter changes in the elements. Link et al have reported some success in also refining the mesh density of a FE model with the updating of parameters to better localise damage detection [33].

Response based approaches for assessing the changes in modal parameters has seen a lot of research before finite element modelling became so prevalent. There still is quite a large amount of backing for these methods, with their lower computation
needed compared to model updating approaches. Sensitivity of mode shapes to the introduction of damage to a structure has been discussed earlier and the change in mode shape curvature, being the second derivative of mode shape was first investigated by Pandey et al[17]. Pandey used a finite element model of a simply supported and cantilever beam with simulated cracks of varying depth. It was shown that the change in curvature due to the discrete change in cross section at the point of damage produced a large change in modal curvature due to the relationship

\[ M_{\text{moment}} = EI\kappa \] . Detection using the mode shape curvatures by Pandey et al showed damage above 25% of the cross section was able to be detected with good accuracy. Abdel et al investigated the use of monitoring changes in modal curvatures in a real structure, a quantitative numerical value was also developed to weight the change of curvature in each mode measured, as it is evident that the location of faults needs to be measured over a number of mode shapes, as the location of faults may lie at points of zero curvature in the mode. The developed numerical value was the Curvature Damage Factor, CDF, given by

\[ CDF = \frac{1}{N} \sum_{n=1}^{N} |\nu''_{\text{oi}} - \nu''_{\text{di}}| \] where \( \nu''_{\text{oi}} \) and \( \nu''_{\text{di}} \) are the curvature for the undamaged and damaged cause respectively. It was shown that spurious spikes were shown in the modal curvatures due to measurement noise and the central difference curvature measurement technique for the where there is zero curvature in each mode shape.

Due to noise and measurement uncertainties in mode shapes, taking the second derivative of this mode shape increases the effect noise has on the output, therefore ways of smoothing the mode shape measurements has been undertaken by some. Sol et al used a tenth order Lagrange polynomial to smooth the mode shape data, but this
causes the smearing of the location of the data and therefore direct localization of the fault detection is unable to be found [14].

Another method which has been used to reduce the measurement noise which is accentuated in the processing of the mode shapes to obtain the curvature is the direct measurement of the dynamic strain, which is directly related to the curvature through the extreme fibre bending moment of the section, of the structure using long strain fiber gauge, this has had some success in the research undertaken by Schulz et al [13].

Response based methods using the modal curvatures as well as the model based methods using sensitivity based finite element model updating have both shown promises as damage assessment techniques. It would then seem advantageous to combine the two techniques and in fact a study by Abdel et al has done just this [26]. This research found that incorporating the modal curvatures as a response used in the model updating technique did not in fact help in the convergence of a result. The damage was simulated by applying a three point bending stress to a beam therefore damage was spread over a large proportion of the beam. If in fact the damage was more localised such as a single crack the introduction of the modal curvatures as responses of the model updating technique should in fact help convergence.

### 2.2 Summary of Ideas

There exits a wide variety of vibrational parameter assessment techniques used for fault detection in structures, starting at simple comparison of natural frequencies and mode shapes to more sophisticated and computationally rigorous methods.

Direct analytical models of the mode shape of a beam section with a introduced crack from the Euler-Bernoulli equation have been developed. Updating modal model methods have been developed starting at the direct orthogonalisation...
methods to the sensitivity based methods. The greatest amount of recent development and room for further development is in the use of different modal parameters as responses to be updated. Some response methods that were discussed from easiest to the hardest to form as parameters are, natural frequencies, mode shapes, FRF’s and mode shape curvatures.

Some direct use of the responses has shown promise, with the direct use of the change in modal curvatures to locate a local fault within a structure. The accuracy of measurements has had the greatest influence on how much this method can be used, but work into measurement of dynamic strain directly from strain gauges has been very promising in reducing the noise in the modal curvatures.
3 Preliminary Analysis and Theoretical Predictions

Foundational mathematical principles of the experimental structure to be analysed and the methods used in assessing the changes in modal parameters will be presented within this chapter.

The basic classical Euler-Bernoulli beam equation of the beam structure will firstly be presented, for both a preliminary analysis of the natural frequencies and mode shapes, and also such that the forces and boundary conditions presented in the experimental setup are fully understood.

The mathematics behind the modal model updating techniques is first brought up within this chapter and presented in enough detail that the process of model updating and its limitations and advantages can be known and exploited.

Finally the mode shape curvature method of assessing the change in the structure due to the introduction of the local fault will be presented and the basic principles, mechanics and mathematical approach which is used in its evaluation.

3.1 Euler-Bernoulli Beam equation

Calculations of the mode shapes using Euler-Bernoulli theory for a variety of cross sections and boundary conditions are well understood and relatively easily calculated. The Euler-Bernoulli equation will be loosely derived, but for full derivation the reading is directed to Appendix A.

The experimental beam that will be analysed within the thesis is suspended in free-free boundary conditions.

Taking an element of the beam and looking at all the forces on this elemental section, the dynamic equation can be found through equilibrium and continuity, a diagram of the forces and elemental section of the beam is shown in Figure 3.1.
Solving for the forces in the vertical direction and for the bending moments on the elemental section yields,

\[ \rho A(x) dx \frac{\partial^2 u}{\partial t^2}(x,t) = -dV(x,t) \]

\[ dM(x,t) - dx[V(x,t) + \frac{dV}{2}(x,t)] = 0 \]

Assuming plane sections remain plane then substituting,

\[ V = \frac{\partial M}{\partial x} \]

\[ M(x,t) = EI(x) \frac{\partial^3 u}{\partial x^3}(x,t) \] [2]

Gives,

\[ \frac{EI}{\rho A} \frac{\partial^4 u}{\partial x^4}(x,t) + \frac{\partial^3 u}{\partial t^3}(x,t) = 0 \]

Solving this equation by the separation of variables technique and for the following boundary and initial conditions,
$u(x, 0) = u_o(x) - IC1$

$\frac{\partial u}{\partial t}(x, 0) = u_o(x) - IC2$

$M = EI \frac{\partial^2 u}{\partial x^2}(0, t) = 0 - BC1$

$M = EI \frac{\partial^2 u}{\partial x^2}(l, t) = 0 - BC2$

$V = \frac{\partial}{\partial x} [EI \frac{\partial^2 u}{\partial x^2}(0, t)] = 0 - BC3$

$V = \frac{\partial}{\partial x} [EI \frac{\partial^2 u}{\partial x^2}(l, t)] = 0 - BC4$

These above boundary conditions are some of the most important points to be taken from the analysis of the Euler-Bernoulli equation. Boundary conditions show that there is both no bending moment as well as shear at the ends of the beam. Which will become an important factor later in the methods used for analysis of damage within the beam section.

Solving the above equations yields the so called frequency and motion equation respectively as follows.

$$\cos \beta_n L \cos \beta_n L = 1 \quad [3.1]$$

where,

$$\beta_1 L = 4.73$$

$$\beta_2 L = 7.85$$

$$\beta_3 L = 10.996$$

$$\beta_4 L = 14.14$$

$$\omega_n = \frac{(\beta_n L)^2}{2\pi} \left( \frac{EI}{\rho AL^4} \right) \quad [3.2]$$

$$u(x, t) = C_n \left[ \sin \beta_n x + \sinh \beta_n x + a_n (\cos \beta_n x + \cosh \beta_n x) \right]$$

where $a_n = \left( \frac{\sin \beta_n L - \sinh \beta_n L}{\cosh \beta_n L - \cos \beta_n L} \right) \quad [3.3]$
Solving equation [3.2] for the geometric and material properties for the experimental beam of:

\[ E = 200 GPa \]
\[ \rho = 7.8 \text{ kg/m}^3 \]
\[ A = 20 \times 50 \text{mm} \]
\[ I = \frac{50 \times 20^3}{12} \text{mm}^4 \]
\[ L = 1000 \text{mm} \]

Table 3.1 (First four modes natural frequencies using Euler-Bernoulli equation)

<table>
<thead>
<tr>
<th>n</th>
<th>( \omega ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>104.1</td>
</tr>
<tr>
<td>2</td>
<td>286.9</td>
</tr>
<tr>
<td>3</td>
<td>562.6</td>
</tr>
<tr>
<td>4</td>
<td>929.9</td>
</tr>
</tbody>
</table>

The first four bending mode shapes plotted from equation [3.3] can be seen in Figure 3.2.
Chapter 3

3.2 Modal Model Updating

Iterative modal model updating techniques are all based on improving the correlation between analytical and experimental data. The specific modal model updating technique to be discussed below is a sensitivity based model, which updates a finite element model parameters such that there updating is weighted by the structures sensitivity so that the difference between the responses is minimised.

3.2.1 System Discretization

The governing equation for a discrete system is given by,
\[ [M] \ddot{u}(t) + [C] \dot{u}(t) + [k] u(t) = \{f(t)\} \]

This equation is given by the balancing of applied and reactive forces applied to a system.

Generally finite element models neglect any damping due to the complexities it introduces and the inability of adequate methods of measurement so the governing equation of a discretized system reduces to, and will be further only referred to in the case of,
\[ 2([M] \omega^2 + [K]) \{u(t)\} = \{f(t)\} \]

The modal parameters that are to be used in the updating of the finite element model are natural frequencies and mode shapes. To find the natural frequencies and mode shapes of the discretized system equation [3.4] is solved with a forcing function set to zero giving,
\[ ([K] - [M] \omega^2) \{u(t)\} = 0 \]

The non-trivial solution of equation [3.5] is found by solving for its determinant and yields the eigenvalues or natural frequencies.
\[ \det([K] - [M] \omega^2) = 0 \]
Mode shapes are then determined by substitution of the eigenvalues obtained from equation [3.6] back into equation [3.5]. Most finite element packages use a Lanczos Subspace method of solution to reduce the amount of computations in the solution [11]. A quick loose description of the Lanczos method of solution, transforms equation [3.5] into the Lanczos subspace domain and uses the orthogonality with respect to mass properties of the eigenvectors to solve.

Using the orthogonality with respect to mass of the eigenvectors can be shown that equation [3.5] is in fact,

$$[K]\{\phi_j\} = \omega_j^2 [M]\{\phi_j\}$$  \hspace{1cm} [3.7]

### 3.2.2 Sensitivity Calculation

Sensitivity is defined by the slope of the response due to the change in parameter, so they therefore show how much the changes in a specific parameter has on a specific response, it is given generally by,

$$[S] = S_{ij} = \left[ \frac{\partial z_i}{\partial \theta_j} \right]$$  \hspace{1cm} [3.8]

The calculation of the sensitivity matrix for the given responses of natural frequencies and parameter $\theta$ can be found easily from the derivation of equation [3.7] and premultiplying by $\phi_j^T$ [1]

$$\frac{\partial \lambda_j}{\partial \theta} = \phi_j^T \left[ \frac{\partial [K]}{\partial \theta} - \lambda_j \frac{\partial [M]}{\partial \theta} \right] \phi_j$$  \hspace{1cm} [3.9]

It can be seen that the sensitivities for the $j$th eigenvalue and eigenvector are required to calculate the sensitivities of the $j$th eigenvalue. It has also been shown that the calculation of the sensitivities for the $j$th eigenvector as responses for a given parameter are also only dependant the calculation of the $j$th eigenvalue and
eigenvector. The derivation of the sensitivity for the eigenvectors is more complex than for the eigenvalues and the reader should be directed to references [1][11] if derivation is needed.

### 3.2.3 Model updating Algorithm (Bayesian Estimation)

Bayesian parameter estimation uses weighting matrices on both the parameters and responses, and the difference between the estimated and target model are resolved by minimising a weighting error.

In general an error function is minimised so that the target results are, as given below,

\[ \varepsilon = \delta z - S_j \delta \theta \]  \hspace{1cm} \text{[3.10]}

where,

\[ \delta \theta = \theta - \theta_j \]

\[ \delta z = z_m - z_j \]

\[
\begin{bmatrix}
\text{target results} \\
\text{actual state}
\end{bmatrix}
= 
\begin{bmatrix}
\text{current state} \\
\text{sensitivity Marix}
\end{bmatrix}
\begin{bmatrix}
\text{parameter changes}
\end{bmatrix}
\]

For the more specific case,

\[ \varepsilon = \{ \delta z \}^T [C_R] \{ \delta z \} + \{ \delta \theta \} [C_R] \{ \delta \theta \} \] \hspace{1cm} \text{[3.11]}

This error is then minimised by using the following algorithm,

\[ \{ \theta_j \} = \{ \theta_{j-1} \} + [G] \{ -\delta z \} \] \hspace{1cm} \text{[3.12]}

where,

\[ [G] = ([C_R] + [S]^T [C_R] [S])^{-1} [S]^T [C_R] \] \hspace{1cm} \text{[3.13]}

It can be seen by the general case and the specific solution to the model updating algorithm using Bayesian estimation, that target results are a function of the current state of the finite element model and changes in parameters weighted by the
sensitivity of the system to the change in parameter being updated. The difference is
minimised between the updated analytical model and the target results by the
minimisation of the error function weighted by the confidence in both the parameters
and responses $[C_p]$ and $[C_R]$ respectively [11].

### 3.3 Change in Mode Shape Curvature

After derivation of the Euler-Bernoulli theory for a continuous beam and
transverse vibration, the extension of the mathematical theory behind the derivation of
the mode shape curvature is quite simple.

The mode shape curvature is quite simply the second derivative of mode shape
given in equation [3.3], this yields the curvature of mode shapes being,

$$u^\prime\prime(x,t) = C_{nn} [\sinh \beta_n x - \sin \beta_n x + a_n (\cosh \beta_n x - \cos \beta_n x)]$$

From equation [3.14] it can be seen that for the boundary conditions of the
free-free beam that the curvature is in the same form as that of the mode shapes with
the exception of the end conditions where it can be seen that the curvature is zero at
the end points, the first four curvatures of mode shapes can be seen in Figure 3.3.

One last equation which must be tackled to fully understand the mathematical
background for this analysis method of assessing the change in modal parameters, is
the bending moment curvature equation [3],

$$M_{\text{moment}} = EI \kappa$$

From equation [3.14] it is seen that the curvature is continuous also for
continuity to hold the bending moment must be continuous across all sections on the
beam, also the beam is homogenous so Young’s Modulus, $E$, is also continuous
across the beam. At the point of simulated crack damage the cross sectional area is
reduced and therefore reduces the moment of inertia, $I$, discretely. This is the basis
of this method of using the change in mode shape curvature due to the change in modal parameters at the location of simulated damage.

Figure 3.3 (First four analytically derived bending mode Curvatures)
4 Experimental Modal Analysis

Modal analysis is the determination of any or all of a system's modal parameters of, natural frequencies, damping ratios and mode shapes etc, through experimental measurement over a grid of points across a structure surface. The experimental procedure of modal analysis is based upon the measurement of the vibrational output of a system due to a known input vibration source. The peaks in the output signal at a certain frequency is generally responsible for a structure's resonance and the output phase changes 180 degrees from the input signal phase. From the measured modal parameters, then using appropriate analysis of the output a mathematical expression for each parameter and a mathematical model for the entire system can be developed to describe and predict the system's response.

In general the layout of a test setup for excitation of a structure and the data capture and processing is shown in Figure 4.1.
4.1 Test setup

4.1.1 Beam Properties and dimensions

The beam that was chosen for the experimental analysis to be undertaken on was a mild steel beam with a rectangular cross section, as apposed to a square section so that the bending modes in the two transverse directions are not coupled. The properties and dimensions of the beam are as follows,

\[ E = 200 \text{GPa} \]
\[ \rho = 7.8 \text{kg/m}^3 \]
\[ \text{Length} = 1000 \text{mm} \]
\[ \text{Cross Section} = 20 \times 50 \text{mm} \]
\[ I = 3.33 \times 10^4 \text{mm}^4 \]

The beam is also analysed such that the transverse bending modes are measured in the direction of the shortest side of the cross section. This also helps to reduce any ability of contamination of measurements from the other transverse mode shape direction, as in this configuration the bending mode is more sensitive to the excitation.

4.1.2 Crack Location and dimensions

The location and depth of the simulated crack was chosen for specific reasons. As the simulated crack introduces a local reduction in stiffness and cross section, then the greatest changes in modal parameters will happen when the location of the simulated crack is at a point of maximum bending moment. This can be intuitively understood, by noting the bending moment curvature equation [3.15] and Figure 3.3. Also the sensitivity of the beam shown in Figure 6.5 shows that the introduction of the simulated crack would cause the greatest change in modal parameters if its location is within 15% of either end. The initial crack was located just off centre at a distance of

Gareth Forbes  4.26
540mm from the datum end. The location of the crack was also not to be located at a point of symmetry as this would not allow the comparison of the results for using the natural frequencies only as responses, and the introduction of the mode shapes as responses for the modal model updating method, this will be discussed further in chapter 6. The initial depth of the simulated crack was to be 25% of the depth of the cross section, 5mm, as all previous analysis methods discussed in journal articles reviewed in the Literature review of crack detection had shown that depths below 20% of cross-section were not able to be measured very accurately. The initial crack at 540mm from the datum end, was then extended to 50% of cross section, such that the comparison of the increase in damage could be assessed.

A second crack was introduced at 190mm from the datum end for two reasons, the first being that the methods used to asses the change in modal parameters could be evaluated in there ability to locate two regions of localised damage. Secondly it was also placed at 190mm from the datum end so that it lay within a less sensitive area of the beam for the first four bending modes. The location of both cracks are indicated in Figure 4.2.
The width of the simulated crack was kept as small as possible, approximately 0.4mm, for the first introduced crack, so that it would as closely model a real crack, but still remain as an open crack, so that the linearity of the system was not compromised. Normally the cracks which develop in high speed machinery are fatigue cracks with breathing behaviour, not as open cracks, however varying stresses on a structure with also statically stressed areas can cause open cracks, so the use of an open crack simulation is a physically possible scenario [25].

The second crack had a much wider width, closer to 2mm, this was due predominately to the fact that the equipment available for introducing the second wider crack was readily available within the school and was cut with a circular bandsaw, where as the initial crack had to be outsourced and was cut with a oil bathed wire cutter.

A close up look of the introduced simulated cracks, and the difference between the two relative widths of the 04mm and 2mm crack can be seen in Figure 4.3.

![Image of simulated cracks](image)

**Figure 4.3 (0.4 and 2mm simulated cracks)**

### 4.1.3 Boundary Conditions

Testing of the beam structure was undertaken with simulated free-free boundary conditions. The reason for choosing free-free boundary conditions is that
simulating free-free conditions is done to a much higher degree of accuracy than is usually achieved for other common boundary conditions, such as built in ends or simply supported end conditions.

To simulate the free-free boundary conditions, the beam was supported by soft springs. It is widely accepted that if the period of free body motion due to the new spring mass system which the constraints introduce, is less than an order of magnitude below the lowest natural frequency of the supported structure, the boundary conditions will impact negligibly on the results [1][2]. The observed period of the supported beam in free body motion was approximately 3Hz, which is approximately 30 times less than the lowest natural frequency of the beam. It can be concluded that the assumption of idealised free-free boundary conditions should have no significant effect on the measurements and assumption of idealised free-free boundary conditions.

The entire experimental setup, with the application of the forced input (1), spring supports (2) and approximate locations of the simulated cracks (3a)(3b) can be seen in Figure 4.4.

Figure 4.4 (Experimental Setup)
4.2 Processing/Data Capture Software (ACE Signal Calc.)

Data physics’ Ace Signal Calc. is a two channel signal processing interface, which can record and perform real time and store analysis data for a single input and output signal [8]. The processing of these signals is also able to be done in the Ace platform with the generation of Frequency Response Functions, FRF, Coherence and signal averaging is able to be done within the Ace systems’ software interface. The Ace system is able to perform a wide range of post processing features along with the ones that will be used mentioned above. Interpretation of the captured data and the signal processing performed can be viewed and used for analysis within Ace system or the exporting of this data is also available, with compatible formats available for most widely used post processing software.

4.3 Excitation Method and Sensor Location

4.3.1 Excitation Method/Signal

Two main methods of introducing an excitation signal to the system so that the vibrational parameters of the system can be measured are available. Being excitation of the system with either an impulse from a hammer or a processed signal applied through an electrodynamic shaker. The amount of energy able to be input into a system is a function of the applied force and the time it is applied. It is therefore evident that the amount of energy able to be input into the system from an impulse from a hammer excitation is much less than is put into the system by an applied processed signal from an electrodynamic shaker. It was therefore decided that for a deflection large enough to capture the change in modal parameters and the influence of the local simulated crack on these, then the shaker input method with its larger ability to apply energy to the system was chosen.
Now that an applied signal is to be input through a shaker, the appropriate signal input is to be chosen. A broadband signal input was chosen for its ability to excite a large range of frequencies over a short record length. From the large number of broadband excitation signals that are widely used, pseudorandom excitation was chosen. Without going into in deep detail as to the subtleties of each kind of excitation signal, it is suffice to say pseudorandom excitation is a random signal which repeats exactly every record length, so that it is in fact a periodic random signal. This means that no windowing function needs to be applied as that aliasing does not occur and a rectangular window function is used. Pseudorandom signals also have the property that theoretically no averaging needs to be taken across an ensemble of signal inputs. The advantage that pseudorandom signals have over pure random signals, means that no aliasing, leakage or resolution bias error is present in the data. The disadvantage of pseudorandom input signals is that any non-linearities within the system are excited the exact same way with each record length such that they cannot be averaged out, as can be done with a pure random signal.

### 4.3.2 Excitation and sensor location

Effects that all transducer have on the system need to be taken into account when taking measurements and choosing what kind of excitation should be used [34]. For shaker excitation the force transducer is attached to the structure which will cause a localising of mass at the point of attachment, as well as its attachment to the structure should be taken into account. A stinger attachment method was used in this experimental setup to connect the force transducer to the shaker input, which is a slender piece of wire which is stiff in the axial direction and flexible in all other directions, therefore it helps ensure that only one direction of excitation is applied.
The location of the excitation signal should be applied at a point upon the structure which is not a node point at any frequency if possible. Also the points of greatest displacement over the range of mode shapes to be measured provide the best point for applying the input signal as any displacement that the input provides here will have the biggest effect on the output displacement of the system. Calculation of the Normalized Modal Displacements is one of the best ways to find the place of best signal input location. Figure 4.5 shows the Normalized Modal Displacements calculated for the test beam, and it can be seen that the corners of the beam give the greatest Normalized Modal Displacements, so this indicates the best location for the input signal and is in fact the point at which the signal input is applied for the experimental beam. In fact because of the free-free boundary conditions, the corners of the beam are never a nodal point and provides the most ideal sensor location. This phenomenon does not often exist in a real structure however.

**Figure 4.5 (Normalized Modal Displacements)**

Location of the output sensors, accelerometer, for measurement of the output, can be done by measuring the entire output of the system at once, with a number of accelerometers across a grid on the structure. However as only a small amount of
accelerometers are available, a roving accelerometer that is moved over the grid of measurement points is used, taking advantage of the theory of reciprocity in linear system.

A sufficient amount of measurement points in a grid across the surface of the structure must be taken so that mode shapes can be constructed without real, or visual aliasing. A grid of 63 measurement points was decided on, with 21 across the length by 3 deep across the breadth would be sufficient to measure the amount of bending and torsional modes that are sort for analysis without the above mentioned aliasing.

The numbering of the measurement points can be seen in Figure 4.6, this numbering pattern will be referred to later in the text. The location of the 540mm and 190mm crack from the datum end lays between the line of measurements (11,32,53) and (12,31,54), (4,39,45) and (5,38,47) respectively.

Figure 4.6 (Measurement point numbering scheme)
4.4 Measurement Equipment

4.4.1 Input Signal Amplifier

The signal output from the ACE Signal Calc. system is only put out by a laptop PC so is quite weak, therefore amplification of this signal into the shaker to excite the system with a sufficient amount of energy is needed. A Ling Dynamic Systems PA25E amplifier was used for this signal amplification. The PA25E is a linear amplifier which provides low noise and low distortion performance. With a useful range of 10Hz-10kHz.

4.4.2 Shaker

Application of the signal to the structure through a shaker input was chosen, the shaker used in the test setup was a Ling Dynamic Systems permanent magnet shaker V203, with a useful range of use of 5Hz-13kHz.

4.4.3 Force Transducer

Once the excitation signal has been applied to the system, it must then be measured at its point of application, as its not sufficient use the signal as it was when it left the signal generator, as it will change as it goes through the transfer functions of all the system parameters on the way to being applied to the structure. The force transducer that was used to measure the actual input signal to the structure was a Bruel and Kjaer Force Transducer 8200. The force transducer works by having a piezoelectric delta shear structure inside, and when deflected will cause a charge to flow in proportion to the applied force due to the piezoelectric effect, such that it can measure the input force.
4.4.4 Accelerometer

To measure the output of the structure due to the input of the excitation signal a roving accelerometer is used to measure the acceleration at a number of points across the structure. A Bruel and Kjaer 4393, with a mass of 2.5 grams accelerometer is used to measure the acceleration response. The accelerometer also has an electrical output caused by the deflection of the delta shear piezoelectric structure within the accelerometer which is deflected under the force caused by the acceleration of the structure at the measurement point, and as the force is proportional acceleration, and acceleration output is able to be measured.

4.4.5 Output Charge Amplifier

The last piece of equipment in the measurement of the test structure is the charge amplifier of the output signal. As the output from the accelerometer is from the piezoelectric charge produced from the deflection of the delta shear transducer under acceleration, it is understandable that its output is very low. Therefore, the output needs to be amplified before any analysis can be done on the signal. A Bruel and Kjaer 2635 charge amplifier is used to amplify the accelerometer output.

The Bruel and Kjaer 2635 charge amplifier is a four stage amplifier consisting of an input amplifier, low-pass filter-amplifier, integrator amplifier, and output amplifier.

4.5 Interpretation of Frequency Domain Results

Once the appropriate signal has been applied to the structure and the output signal has been able to be obtained then the use of these signals to calculate the systems modal parameters can be done.
A large range of response data can be analysed from the experimental modal analysis of the test setup, but the discussion of the interpretation of these responses will be limited to the functions used within the thesis and described below, of Frequency Response Function and Coherence function.

### 4.5.1 Frequency Response Functions

Ideally the measurements of both the input and output signals should be sufficient to calculate the systems frequency response, however in reality the system has noise in the form of mechanical structure irregularities and electrical distortions in the signal. There is also distortion to the signal through the limited analysis resolution. So in order to calculate the ratio between the input and output of the structure statistical functions need to be employed to account for this noise in the signals.

\[ H_1(\omega) \] is an estimate of the Transfer function by dividing the cross-spectrum by the auto-spectrum given by \[ H_1(\omega) = \frac{G_{xx}(\omega)}{G_{xx}(\omega)} \], and is the best estimate for noise in the output. The cross-spectrum is the ensemble average of the spectrum of the output signal relative to the input signal and the auto-spectrum is the ensemble average of the spectrum for the input signal.

\[ H_2(\omega) = \frac{G_{xx}(\omega)}{G_{xx}(\omega)} \] is another estimate for the FRF but it minimises the effect of noise on the input to the system, this can occur as at resonance peaks the structure is very compliant to the input signal so that all the force is taken up accelerating the excitation system and very little force is put into the structure itself so the measured input form the force transducer will be dominated by noise. \[ H_2(\omega) \] is calculated by dividing the auto-spectrum of the ensemble average of output signal by the ensemble average of the cross-spectrum between the input and output signal. The \[ H_1 \] estimator
was used in the measurements taken in the experimental modal analysis of the beam, as it was seen that the most amount of noise was in the output signal, due to the accelerometer being placed close to nodal points at some measurement points, and the input signal was relatively noise free due to the large input force and excitation position.

Figure 4.7 shows a set of typical FRF measurements for the structure. The measurement points displayed are long one edge of the beam at measurement locations 1, 11 and 21 respectively for the beam without any introduced cracks.

It is evident that the location of the poles, or natural frequencies are equal in frequency for all measurements, with the first four poles being the first four bending modes, the fifth pole being the first torsional mode.

The general nature of these FRF’s should be noted to asses in a preliminary manner if the measurements are giving valid results. The first measurement point is the driving point and, as discussed earlier, will not be a node for any mode, so therefore all measurement peaks should be sharp and distinct and have an accompanying zero as is evident.

The second FRF corresponding to the eleventh measurement point is mid way along the structure and lays on the point of symmetry for all even bending modes and odd torsional modes, so should have very much reduced poles at this location, due to it being on a node location for these modes. This is especially evident for the second bending mode, where the peak is basically not visible, and very much within the area of the signal noise dominated region. The later even modes, which should also have no peak due to the measurement point being a nodal location, but in fact show slight peaks, is most likely due to the increased curvature at this location for higher modes,
and due to the accelerometers width across the measurement point, the rotational inertia becomes more dominant and shows a measurement peak.

The last FRF in Figure 4.7, is for measurement point twenty-two which is on the same side as the applied signal but opposite end. Being a significant distance from the point of application of the excitation signal, there should be little zeros present in the signal, as is seen. If the measurement was in fact in the diagonal opposite corner no zero values should be present but due to the location being on the same side of the beam as the excitation signal, the torsional mode peaks will still have corresponding zeros which is show in the only two zeros in the signal occurring after the first and second torsional mode peak locations.

Figure 4.7 (FRF’s for measurement points 1, 11, 21, for beam without crack)
4.5.2 Coherence Function

The coherence function, $\gamma(\omega)^2$, is a measure of the system’s linearity or repeatability. The coherence is calculated by taking the statistical correlation between two variables

$$\gamma(\omega)^2 = \frac{|G_{xy}(\omega)|^2}{G_{XX}(\omega)G_{FF}(\omega)}.$$ 

Therefore, if the relationship between the input and output is not independent, i.e., linearly related, then a coherence of 1 should be obtained. So it can be seen that observing the coherence across the measurements will show the measurements conformance to a linear system and is a tool to evaluate any measurements accuracy.

The coherence of the three measurement points 1, 11 and 21 without a simulated crack are again used as examples of typical results obtained for the beam shown in Figure 4.8, with the coherence show below the FRF for which it was measured. The coherence of the measurement points one and twenty-one are very good for all frequencies with only the sharpest of spikes in non-linearity at the location of the zeros of the corresponding FRF function. With values of coherence of very nearly 1 for all other frequencies showing the system is very linear under the conditions at these points of measurement.

The coherence for the eleventh measurement point which was outlined before, lies on the nodal lines for all even bending modes and odd torsional modes, shows a coherence much lower in value at more locations than just at the zeros of the corresponding FRF. It is seen that the coherence is very low for the position of the poles, or peaks, for the even bending modes, this is due to the very low output signal obtained here, and therefore non-linearity’s dominate the FRF at these frequencies at then location. Importantly however the coherence is very good and very close to 1 at and around the odd bending mode peaks in the FRF.
The coherence is also a good indicative indicator of how far away single degree of freedom curve fitting bands should be placed when curve fitting for mode shapes around a resonance peak. As any section of high non-linearity of the system will not be a good measurement parameter to fit a linear curve fitting function to.

Figure 4.8 (FRF’s and Coherence for the 1\textsuperscript{st}, 11\textsuperscript{th}, 21\textsuperscript{st} measurement points)
4.6 Modal Parameter Construction from Frequency Domain Data

4.6.1 Modal Parameter Construction Software

The software interface which was used for the construction of the modal parameters from the experimentally obtained frequency data was Vibrant Technology’s ME-scope [9].

ME-scope is a software interface which can display and convert data obtained through signal measurements being FRF measurements into structures modal parameters being the natural frequencies and mode shapes. ME-scope uses curve fitting of FRF data imported into the database as well as shape models correlating the measurement points for the FRF’s to display the mode shapes. Curve fitting FRF’s and extracting modal parameters from the FRF’s and using these parameters to display the mode shapes reduces the data needed for analysis tremendously. The mode shapes generated in ME-scope can be used for direct analysis or can be exported to some other software interface such as Matlab or FEM tools for further analysis.

4.6.2 Curve Fitting of Frequency Response Functions

After the generation of the FRF’s for the structure at all measurement points the generation of the structures modal parameters can be done from the interpretation of the FRF’s.

The construction of the natural frequencies is quite simple with the natural frequency of each mode being given generally by the peak in the FRF’s, were the ratio of excitation to response is locally a maximum. This peak and therefore the natural frequency should be the same for each FRF measured as natural frequency is a global parameter to the system. The natural frequency may in fact change slightly
from each measurement due to the local mass loading of the roving accelerometer, and when the curve fitting of the FRF’s are undertaken it is suggested a local parameter curve fitting method should be used [1][9].

Curve fitting of FRF’s is the analytical process to determine the mathematical parameters which give the closest possible fit to the measured data [7]. In general the global curve fitting fits the peaks, or poles, from each FRF in a least squares sense and fixes this for all measured FRF’s then fits the residues for each measured FRF using the global pole location [7][9]. Local curve fitting methods use the peak or pole location for each mode in each measured FRF to measure the residual for that measurement. This fact means that the pole location is allowed to vary for each measurement and can lead to spurious modes being calculated or purely local modes, which corrupt the data if careful attention is not taken in reviewing the processed data [7][9]. Damping is measured the same way for each form of curve fitting by measuring the value of the fitted FRF at the half power points, \( \omega_j^{(1)} \) and \( \omega_j^{(2)} \), which lie other side of the natural frequency \( \omega_j \) and utilising the relationship [2],

\[
\bar{\zeta}_j = \frac{\omega_j^{(2)} - \omega_j^{(1)}}{2\omega_j}
\]  

[4.1]

Global polynomial curve fitting is the most common and adaptable method of curve fitting, however due to the very light damping of the beam structure and the local mass loading of the accelerometer, global polynomial curve fitting caused the mode shapes derived to show non-physically viable behaviour, this can be seen in the fourth bending mode shape generated in Figure 4.9 using this method. Further discussion and validation as to why the global-polynomial curve fitting method was
unable to construct the mode shapes for the shaker excited beam setup adequately can be found in Appendix C.

![Figure 4.9 (Fourth bending mode shape using global-polynomial curve fitting)](image)

Figure 4.9 (Fourth bending mode shape using global-polynomial curve fitting)

Curve fitting using Quadrature picking methods however over came this problem and allowed the construction of the mode shapes accurately. Quadrature picking constructs the mode shapes just based on the imaginary component of the FRF at each measurement point. The direction of the imaginary component of the FRF at the measurement point gives the relative phase, of either in phase or out of phase to the driving point FRF measurement. The magnitude of the imaginary component then gives the relative displacement of the measurement point with relation to the driving point.

Quadrature picking exploits the phenomenon that at resonance peaks the spring force and mass force in the equilibrium equation of motion, equation \[3.4\], are equal to one another. This means that any import force is directly equal to the damping of the system and therefore produces an FRF which is purely imaginary, and either in phase or out of phase to the driving force. As Quadrature picking only uses the values of the FRF functions at the point of a resonance peak and not fitting by an
equivalent single or multi-degree polynomial then there is no estimation of damping, and only normal modes can be found in the mode shape estimation using this technique.

### 4.6.3 Interpretation of Mode Shapes

After the appropriate estimation technique for fitting the mode shapes to the measured FRF data, then interpretation of the mode shapes needs to be done to assess their quality and ability to be used for later analysis. It was discussed earlier that when observing mode shapes a strong understanding of the physical ability of the system actually modelled for the estimated mode shape. Therefore if shapes that show unrealistic deflection shapes, such as large changes between measurement points [9], then this is evident of poor curve fitting. As this system is continuous and homogenous any deflection should also display these attributes of continuity and homogeneity. If the mode shapes do not conform to these above conditions they should not be relied upon, and further investigation should be sort, as was described in section 4.6.2.

Eight mode shapes in total are measured during the modal analysis of the structure and can be seen in ascending frequency order in Figure 4.10 and Figure 4.11. The first four bending modes for the beam without any introduced crack, constructed using Quadrature picking methods from the measured FRF’s can be seen in Figure 4.10. It is clear that these mode shapes show the properties of being physically viable shapes, continuous and homogenous in nature. These mode shapes are used as the basis of the analysis methods for assessing the change in modal parameters of the structure due to the introduction of the simulated crack.
Figure 4.11 shows the fifth and sixth bending modes, and the first and second torsional modes. These modes are separated from the others as they will not be used in the methods of assessing the change in modal parameters due to the introduction of the simulated crack. The reasoning behind this for the fifth and sixth bending modes can be seen for the sixth bending mode here, where the coupling to the second torsional mode can be seen with its both bending and twisting motion. This happens due to the proximity of the frequency of the sixth bending mode and the second torsional mode. Later measurements with the introduction of the simulated cracks also saw the degradation in the quality of the fifth bending mode shape, so it was also omitted in its use of the assessment of change in modal parameters.

The quality of the two measured torsional modes is really quite good as is seen in Figure 4.11, and although there quality of measurement degrades a little for the later measurements with the introduction of the simulated crack, this is not the predominant reason for there omission to the analysis techniques. The ability of the
finite element model to converge on results for the torsional modes of vibration is why they were decided to not be used for the change in parameter analysis techniques and is discussed further in section 5.2.2.

The mode shapes constructed for all the measurement cases, are not shown, for report brevity, but they resemble the same generalised shape closely for all measurements and damage conditions to the non-damaged mode shapes in Figure 4.10 and Figure 4.11.

Figure 4.11 (Fifth, sixth bending modes; first, second torsional modes, without crack)
5 FEA Modelling

Finite element modelling takes on two roles within the thesis. Firstly a finite element model is constructed such that it can be used in the Modal Model Updating technique, as it is the basis of this technique that a finite element model is altered in such a way, that the output result differences between the finite element and measured output are minimised.

The second role that Finite Element Analysis takes within the thesis, is to provide the simulation of the experimental results of the damaged beam. This is used for validation of each assessment method, as the true ability of the method with out the introduction of measurement noise can be undertaken. Finite Element modelling also allows for initial estimation of the results expected during measurements, and it is this reason why finite element analysis has seen such wide spread adoption across the engineering field.

Within this chapter all the fascists of constructing the finite element models and for use within the analysis and simulation of the results will be shown, as well as a discussion of the software used and the preliminary results obtained for the experimental model without a simulated crack.

5.1 Pre/Post processing Software

The pre/post processing software used for the Finite Element Modelling of the beam structure for the model updating method, and FEA simulation of experimental results was MSC Patran [12].

Patran is an open, expandable pre and post processing system where FE modelling can be undertaken and results accessed. This can involve the importing of CAD models into Patran for meshing or the creation of the entire FE model within
Patran ready for analysis. Applied forces and boundary conditions are also applied in the Patran environment before analysis of the FE model can be calculated by some other program. The computational results obtained by solving of the applied conditions to a FE model can then be accessed in Patran and displayed through the interactive graphical interface. Patran is there to facilitate the user in interpreting the results obtained from the numerical data that is produced from the solving of the equilibrium and continuity equations in other software platforms.

5.2 Mesh Generation

The fundamental theory behind finite element analysis is the discretization of a continuous system into well understood piecewise fundamental elements and equations. The matrix equation of motion for a discretized system is given in equation [3.5]. The biggest decisions that need to be made in the discretization of a geometric model of a structure into mathematically defined structure, is the element choice, and mesh density or refinement.

5.2.1 Element Choice

Two predominant forms of meshing techniques are available in most commercial FEA packages. These are isometric and parametric meshing, with the meshing patterns being completely ordered with quadrilateral geometric shapes or parametrically defined tetrahedral shaped mesh, respectively. Iso-meshing procedures are generally more stable, however the increase in popularity of tetrahedral meshing forms have increased stability and there wide spread use [4], however Iso-meshing was used for the meshing of beam structure.

As a three-dimensional model was to be constructed, an element choice from the three-dimensional Iso-meshing element choices needs to be done. HEX 20 [12]
elements were chosen for the meshing of the three-dimensional beam due to their increased ability to model bending and reduce shear locking in the elements [4].

5.2.2 Mesh Refinement

Refinement of the mesh density across the structure needs to be done so that the results obtained accurately model the actual results. The amount of nodes, such that shape functions for the inter element connections can sufficiently converge to the actual underlying deflection shape, is what defines how dense a mesh needs to be to converge to the actual results.

Many methods of assessing results convergence are available and the reader is directed to reference [4] for further reading. The convergence of the results for the first four bending modes and the first torsional mode can be seen in Table 5.1, with \( h \) being the global edge length of the element size with the global edge length halved for each refinement such that \( h_1 = 1 \) through to \( h_3 = 0.25 \). It is clearly evident that the results for the first four bending modes have converged to within less than 0.5% for the first four bending modes. Convergence of the torsional mode however has not happened, and much more refinement of element density across the width of the beam would be needed. This inability for the convergence of the torsional mode in the FEA model without having to construct a much more dense model, meant that the torsional modes were omitted from the analysis techniques for measuring the change in modal parameters to detect the structural damage.

Creation of a densely meshed model could be constructed and then the degrees of freedoms reduced, using degree of freedom reduction [4], so that the model with the same amount of degrees of freedom as the less dense model but with the convergence of the torsional modes so that they could be used within the analysis.
methods could be accomplished. Available time did not permit this, however future work may want to explore the use of this reduction in degree-of-freedom technique.

The final model was chosen with a value of $h_2$ with a global edge length of 50mm which corresponded to the same density as the measurements points. This was chosen due to the adequate convergence of results, and for the coincidence of both nodal points and measurement points which is advantageous in the Modal Model Updating technique of assessing the change in modal parameters due to the simulated damage.

<table>
<thead>
<tr>
<th>Table 5.1 (Convergence of FEA results)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{1b} (Hz)$</td>
</tr>
<tr>
<td>$h_1$</td>
</tr>
<tr>
<td>$h_2$</td>
</tr>
<tr>
<td>$h_3$</td>
</tr>
</tbody>
</table>

5.3 Pre-Experimental Analysis

Just as the use of classical Euler-Bernoulli beam theory can be used to evaluate the modal parameters for a structure before any experimental analysis takes place, or in fact before the structure is constructed, finite element analysis offers a much wider ability to model structures and compute how it will behave in reality and is what has brought about such a popular use of this as an analysis tool.

An initial model of the experimental beam was constructed using FEA, and the results for the first four bending mode shapes can be seen in Figure 5.1, and the comparison of the frequencies calculated for the undamaged beam using FEA and Euler-Bernoulli theory is shown in Table 5.2.
Table 5.2 (Comparison frequencies using FEA and Euler-Bernoulli theory)

<table>
<thead>
<tr>
<th></th>
<th>$f_{1b}(Hz)$</th>
<th>$f_{2b}(Hz)$</th>
<th>$f_{3b}(Hz)$</th>
<th>$f_{4b}(Hz)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-B Theory</td>
<td>104.1</td>
<td>286.9</td>
<td>562.6</td>
<td>929.9</td>
</tr>
<tr>
<td>FEA</td>
<td>103.69</td>
<td>284.52</td>
<td>554.58</td>
<td>910.36</td>
</tr>
</tbody>
</table>

Figure 5.1 (First Four Bending Modes constructed in FEA)

The differences between the finite element results and those calculated using Euler-Bernoulli theory is due to the fact that Euler-Bernoulli theory is a simplification of reality and assumes that plane sections remain plane during deflection, that is there is no “flapping” of the beam across its width when deflected. As the finite element results are not limited to Euler-Bernoulli theory therefore the results obtained from the finite element analysis could be argued with some rigour that they are more valid than the Euler-Bernoulli theory estimated results.

Along with the initial modelling of the undamaged beam, simulation of the introduced crack was modelled in FEA to both assess the ability of the methods used but to also predict the resultant outcomes. The next section will outline the modelling of the crack procedure.
5.4 Simulated Crack Modelling

Simulation of the introduced crack in the experimental beam was done for a Finite Element model. The crack was simulated by a 2mm wide section of the solid beam model being left out at the location of the crack. This can be seen in Figure 5.2, the mesh density can also be seen up to and along the crack section. The mesh can be seen to only be one element thick along the crack model section and having a mesh density of one element per 50mm up until that section. This gives a change in mesh density at the location of the crack 1:25, this is not ideal as a change in mesh density of less than 1:10 at any point on a structure should be avoided, otherwise ill conditioning of the FEA model might exist [4]. The convergence of the results however suggest that the modelling is adequate, but any further analysis might look at improving the modelling of the simulated crack section.

![Simulated Crack Modelled in FEA](image.png)

**Figure 5.2 (Simulated Crack Modelled in FEA)**
Chapter 6

6 Modal Model Updating Analysis Method

Modal model updating has become a widely used technique in structural analysis problems since the early 1990’s. Model updating brings together the strengths of experimental measurement techniques to correct the assumptions taken in analytical modelling. The software used for the updating process is presented generally in this chapter with a more detailed critical review in Appendix D. All the considerations which need to be taken before updating can occur are discussed in enough detail to replicate the results obtained and the justifications behind them. The results are presented with the measures of model updating quality assessment within the results and discussion. Finally the results are discussed and conclusions for the model updating procedure for detection of a simulated crack are detailed.

6.1 Model Updating Software

A proprietary sensitivity based model updating software package called FEM-Tools was used in this method of assessing the change in modal parameters due to the introduction of the local crack.

FEM-Tools is a software package which brings together both experimental and analytical model results with correlation between experimental and FEA modal model data. Updating of the FEA model is able to be done to match the results obtained in the experimental results by modifying the modal parameters with iterative model updating available such that the selected modal parameters are updated until the correlation criterion for the predicted and reference models meet a specified convergence level. Many correlation methods are available in FEM-Tools, such as MAC, COMAC, direct visual comparisons of mode shapes and many more.
6.2 Comparing Numerical Data (FEA) with Test Results

Comparison of FEA data with the experimental test results needs to firstly be undertaken before any model updating algorithm can be run. Equivalencing of the same corresponding mode shapes in both the analytical and experimental mode shapes as well as the corresponding node points to the experimental results and analytical model needs to take place. Specific methods to compare global measures of correlation between experimental and analytical results are also presented.

6.2.1 Mode Shape Pairing

Mode shape pairing must be done before model updating can take place, so that the mode shapes correspond to the same mode shapes in both the analytical model and the experimental results.

A conventional analysis tool for measuring the correlation between any mode shape, used for predominately for finding what mode shapes are equivalent in both the finite element model and the experimentally measured mode shapes, is the calculation of the Modal Assurance Criterion, MAC. The need for this procedure is brought about by the large amount of mode shapes created with finite element analysis and measured results but they do not necessarily correlate in sequential order so a method of determining which mode shapes should be used to update against each other is needed. This is where a global correlation method such as the MAC, is used.

\[
MAC^{(a,e)} = \frac{\left( \sum_{j=1}^{n} \phi_{aj}^e \phi_{ej}^a \right)^2}{\sum_{j=1}^{n} \left( \phi_{aj}^e \right)^2 \sum_{j=1}^{n} \left( \phi_{ej}^a \right)^2}
\]  

[6.1]

where ‘a’ and ‘e’ represent the analytical and experimental mode shapes respectively.
The MAC is also often used as global method of assessing whether damage has occurred within a structure.

### 6.2.2 Node Point Pairing

Pairing of the nodal points of the Finite Element model nodes to the measurement points on the experimental structure must be done so that the response values for the nodes of the Finite Element model can be updated to the values of the measured results. This is done by overlaying the Finite Element model and the geometric positions of the measured data. A geometrical tolerance is then specified and the closest nodes under this tolerance to the measurement points are then updated with respect to that measurement point. As small deviations are to be updated from the experimental results, the mesh needs to be quite dense within the region of the measurement points such that a large distance does not occur between the nodes and measurement points and increase the inaccuracies in the model updating. Another method of assuring the highest amount of accuracy is obtained in the nodal pairing is constructing a Finite Element model with nodes directly corresponding to the measurement points. This method is employed in the Finite Element model of the experimental beam with a mesh generated in the Finite Element model equal to the measurement grid. This ensures that the nodes and measurement points directly correlate.

### 6.3 Calibration of Initial FEA Model

Initially the Finite Element model frequencies differ from that of the experimentally measured ones, as seen in Table 6.1, so for detection of the crack location due to the change in modal parameters to be determined, the initial
differences must be accounted for. This is done by the calibration of the initial Finite
Element model.

Table 6.1 (Initial frequency difference between FEA and EMA)

<table>
<thead>
<tr>
<th>Mode Shape Pair</th>
<th>FEA(Hz)</th>
<th>EMA(Hz)</th>
<th>Diff(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>104.0</td>
<td>104.7</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>286.5</td>
<td>287.5</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>561.2</td>
<td>562.5</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>926.9</td>
<td>931.3</td>
<td>0.05</td>
</tr>
</tbody>
</table>

It is assumed that the density of the beam is known to a high degree of
accuracy and the Young’s Modulus used for the mild steel beam obtained from
general material property data sheets is the most unknown material parameter in the
FE model. The homogeneity of the beam without the introduction of a crack means
that the material properties should be held constant across the section, therefore the
Young’s Modulus is used as the updating parameter in a global sense. The first four
bending modes are the used as the updating responses.
### Table 6.2 (Updated results for beam without crack)

<table>
<thead>
<tr>
<th>Mode Shape Pair</th>
<th>FEA (Hz)</th>
<th>EMA (Hz)</th>
<th>Diff (%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>104.5</td>
<td>104.7</td>
<td>-0.22</td>
<td>99.6</td>
</tr>
<tr>
<td>2</td>
<td>287.7</td>
<td>287.5</td>
<td>0.08</td>
<td>99.8</td>
</tr>
<tr>
<td>3</td>
<td>563.6</td>
<td>562.5</td>
<td>0.19</td>
<td>99.7</td>
</tr>
<tr>
<td>4</td>
<td>930.7</td>
<td>931.3</td>
<td>-0.05</td>
<td>98.5</td>
</tr>
<tr>
<td>5</td>
<td>1185.1</td>
<td>1014.1</td>
<td>16.87</td>
<td>97.4</td>
</tr>
<tr>
<td>6</td>
<td>1389</td>
<td>1364</td>
<td>1.83</td>
<td>97.6</td>
</tr>
<tr>
<td>7</td>
<td>1938.2</td>
<td>1946.9</td>
<td>-0.45</td>
<td>89.9</td>
</tr>
</tbody>
</table>

### Table 6.3 (Updated Parameters for beam without crack)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Old</th>
<th>Actual</th>
<th>Diff (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global E (GPa)</td>
<td>200</td>
<td>2.017</td>
<td>1</td>
</tr>
</tbody>
</table>

#### 6.4 Updating for Detection of Crack

Sensitivity based model updating methods have a wide range of use within in engineering, to help predict and validate various kinds of results. The updating procedure is used within this thesis in such a fashion so that the largest change in parameters that occur within the structure will highlight the presence and location of a local simulated crack.

For model updating procedures there are three main variables which must be defined before the updating algorithm can be solved. Observance of equation [3.11], shows that these variables are the choice of parameters, responses and their respective
weighting functions. The method used for the choice of these variables will be discussed within this section.

### 6.4.1 Choice of updating Parameters

There exists an infinite theoretical choice of updating parameters, however there is only a limited number of parameters available for use in the updating algorithm within the FEM-Tools software package, although there is quite an extensive selection. As it has been discussed in earlier chapters, the introduction of the local simulated crack in the beam structure will cause a reduced stiffness at its location. Therefore it was decided that the best parameter to be used for the updating of the structure would be the Young’s Modulus of the Finite Elements, as the Young’s Modulus is directly proportional to the dynamic stiffness of the structure.

The Young’s Modulus parameters where chosen to be allowed to vary for every element within the finite element model such that the greatest ability to locate the location of the fault to the exact element that it is contained within is available. It is noted however that allowing every element to vary with the parameter chosen is considered the most extreme case of model updating and can lead to ill condoning of the convergence of parameters for larger more complex models [1][24].

Introducing a large number of parameters to update for the model, also causes a non-exclusive solution if the amount of responses that are used to update against are less than the number of parameters. This is due to there being less solutions to the number of independent simultaneous equations which need to be solved than there are variables, so a non-unique solution is then only able to be found.
6.4.2 Choice of Response’s

The choice of Responses to use in the updating algorithm is the most difficult, and important choice that needs to be taken by the user before updating can be performed.

The most common use of updating responses is the use of the natural frequencies, this is predominantly due to the high degree of accuracy that they are able to be measured. Another advantage of natural frequencies as responses is that they are a global parameter to a system so that very few measurements need to be conducted as opposed to the other modal responses which can be used in the updating algorithm. There are instances where more information needs to be introduced which the measurement of other modal responses and there use in the updating process is needed. The mode shapes are often introduced to the updating process as response, as it is widely accepted that the change in mode shapes are much more sensitive to damage introduced within the structure than the change in natural frequencies [1][15][17]. Along with being more sensitive to damage, the mode shapes or some other spatially defined response must be used in the updating method if the structure is symmetric about any axis, for the unique locating of damage [20]. This due to the fact that if the location of some damage about a axis of symmetry and the change it will have on the natural frequency would be the same if it was on either side of the symmetric axis, and therefore smears the parameters updated across the two symmetric element locations. This phenomenon is evident in the results discussed below.

The use of just natural frequencies and both natural frequencies and mode shapes as responses are then used in the updating of the structure, and the results from both methods are then compared. When choosing the mode shape responses a
threshold value of the number of measurement locations which are included is set by finding the biggest difference in displacement between the finite element model and measured modal displacements and then only differences within this threshold value are only then included. The choice of this value is not defined quantitatively and is left up to the speculative view of the user, and the results obtained for different values and the convergence of the parameter changes should be used in deciding on a value. The inclusion of the displacements within 80% of the maximum displacement was chosen for the updating for the results shown, this was found to be the best value, with higher threshold values not allowing enough points to be added for adequate updating, and lower values included more measurement points however more likely contaminated by noise and convergence of results was not then able to be achieved.

The comparison of the FEA mode shapes and EMA mode shapes can be seen in Figure 6.1 with blue and red mode shape displacements representing the FEA and EMA results respectively. It is clear in observing the four bending modes in Figure 6.1, that the end measurement points have a much greater difference in there values than any others, and for the higher order bending modes it is clear that if they were included with a 80% threshold value then no other measurement points would be included in the updating, and it is totally dominated by these measurement points. It was found without the exclusion of these end points convergence of the results was not able to be achieved.

It is quite a common practice to smooth mode shapes or leave out the end measurement points because of the high degree of inaccuracies in the measurement of mode shapes[1][9][10][27].

The use of just frequencies as the responses and then the use of both frequencies and mode shapes as responses are compared for all results in the project.
The use of just frequencies as responses is advantageous due to the ease of measurement, reduction in measurement time, and accuracy of measurement. At first thought it might seem that measurement of the natural frequencies and using in the updating algorithm could not give any spatial information on the location of the local fault, after all the natural frequency can be measured at just one position and is a global parameter of the structure. However once again observing equation [3.11] and equation [3.12], it is seen that the parameter changes are weighted by the sensitivity matrix, which is spatially unique for each natural frequency and as a result of the derivation from the eigenvalues and eigenvectors from the analytical model. This means that the relative change in each natural frequency in fact gives a spatial resolution of the location of the fault. The disadvantages of using just natural frequencies however is that they are relatively insensitive to damage and it is this which has been the driving force for other modal parameters to also be used in the updating process. Another disadvantage of frequency only response updating, comes when a structure has an axis of symmetry, which is quite common in any real structures. It is quite intuitive that if some damage has occurred at a point on a structure, then its effect on the natural frequencies would be equal if the same amount of damage occurred at the equivalent position on the other side of the axis of symmetry. This causes a smearing of the updating parameters across the two locations on opposite sides of the axis of symmetry.
Figure 6.1 (Experimental and FEA Mode Shape Comparisons)
6.4.3 Parameter and Response Function Weighting

Again observing equation [3.11], the parameters and the responses are both weighed by a penalty function. The parameters are weighted by any kind of weighting that the user wishes to specify, but within the FEM-Tools software, they are weighted by an inverse estimated statistical variance. When only one kind of parameter is selected in the updating process, the choice of the weighting function is arbitrary, as it is only this one type of parameter which can be altered for the minimisation of the error function.

The weighting of the response however is a much more important weighting function in the updating configuration used for this project. The weighting of these responses defines to what accuracy the updated system is forced to resemble the chosen responses for adequate convergence of the results. Commonly the weighting suggested for frequencies as responses is a 100% weighting meaning that the results are updated to resemble the target values 100%[1][10]. The weighting for the mode shapes is most often given a much lesser value due to the poorer accuracy in there measurement. A suggested confidence in most measurements and updating procedures is to set a confidence value between 10-30% [1][10]. Once again there is no quantitative reasoning for the choice of the weighting values, and it is best left up to the user observing the results for a range of confidence values and making a judgement on what values they should use in the updating algorithm.

It was found that the best values for the weighting functions, which caused the best locating of the local fault by largest changes in parameters was a weighting of 100% for both frequencies and mode shapes. This large increase in the suggested value of mode shape weighting is able to be done because of the good accuracy that these mode shapes compare to the analytical mode shapes, with the exclusion of the
end points. If the higher mode shapes were to also be used in the updating of the FEA model to locate the fault, a smaller weighting of the mode shapes would however need to be used, due to there larger divergence from the analytical results from the experimental results.

### 6.5 FEA simulated Crack Updating Results

#### 6.5.1 Frequencies only as Responses

![Figure 6.2](Parameter Modifications-frequencies as responses-FEA simulated)

Gareth Forbes
6.5.2 Mode Shapes and Frequencies as Responses

![Parameter Modifications](image1)

Figure 6.3 (Parameter Modifications-mode shapes as responses-FEA simulation)

![Parameter Modification contour for FEA simulation](image2)

Figure 6.4 (Parameter Modification contour for FEA simulation)
6.6 5mm Experimental Crack Updating Results

6.6.1 Frequencies only as Responses

Figure 6.5 (Sensitivities-frequencies as responses-5mm crack)

Figure 6.6 (Parameter Modifications-frequencies as responses-5mm crack)
<table>
<thead>
<tr>
<th>Mode Shape Pairs</th>
<th>FEA(Hz)</th>
<th>EMA(Hz)</th>
<th>Diff(%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>103.2</td>
<td>103.1</td>
<td>0.04</td>
<td>99.5</td>
</tr>
<tr>
<td>2</td>
<td>287.3</td>
<td>287.5</td>
<td>-0.07</td>
<td>99.8</td>
</tr>
<tr>
<td>3</td>
<td>558.2</td>
<td>557.8</td>
<td>0.07</td>
<td>99.4</td>
</tr>
<tr>
<td>4</td>
<td>927.7</td>
<td>928.1</td>
<td>-0.04</td>
<td>98.5</td>
</tr>
<tr>
<td>5</td>
<td>1173.1</td>
<td>1007.8</td>
<td>16.4</td>
<td>97.1</td>
</tr>
<tr>
<td>6</td>
<td>1381.8</td>
<td>1370.3</td>
<td>0.84</td>
<td>98.7</td>
</tr>
<tr>
<td>7</td>
<td>1928.2</td>
<td>1954.7</td>
<td>-1.36</td>
<td>88.6</td>
</tr>
</tbody>
</table>

### 6.6.2 Mode Shapes and Frequencies as Responses

![Normalized Sensitivity](image)

Figure 6.7 (Sensitivities-mode shapes as responses-5mm crack)
Figure 6.8 (Parameter Modifications-mode shapes as responses-5mm crack)

Figure 6.9 (3D Sensitivity Matrix-frequency/mode shape responses-5mm Crack)
### Table 6.5 (Updated results for beam-mode shapes responses-5mm Crack)

<table>
<thead>
<tr>
<th>Mode Shape Pairs</th>
<th>FEA(Hz)</th>
<th>EMA(Hz)</th>
<th>Diff(%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>102.6</td>
<td>103.1</td>
<td>-0.52</td>
<td>99.5</td>
</tr>
<tr>
<td>2</td>
<td>286.3</td>
<td>287.5</td>
<td>-0.42</td>
<td>99.9</td>
</tr>
<tr>
<td>3</td>
<td>554.6</td>
<td>557.8</td>
<td>-0.58</td>
<td>99.4</td>
</tr>
<tr>
<td>4</td>
<td>932.4</td>
<td>928.1</td>
<td>0.46</td>
<td>98.3</td>
</tr>
<tr>
<td>5</td>
<td>1166.9</td>
<td>1007.8</td>
<td>15.79</td>
<td>96.9</td>
</tr>
<tr>
<td>6</td>
<td>1359.2</td>
<td>1370.3</td>
<td>-0.81</td>
<td>98.5</td>
</tr>
</tbody>
</table>

### 6.7 10mm Experimental Crack Updating Results

#### 6.7.1 Frequencies only as Responses

![Figure 6.10 (Sensitivities-frequencies as responses-10mm crack)](image-url)
Figure 6.11 (Parameter Modifications-frequencies as responses-10mm crack)

Table 6.6 (Updated results for beam-frequency responses-10mm Crack)

<table>
<thead>
<tr>
<th>Mode Shape Pairs</th>
<th>FEA(Hz)</th>
<th>EMA(Hz)</th>
<th>Diff(%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>95.4</td>
<td>95.3</td>
<td>0.05</td>
<td>99.6</td>
</tr>
<tr>
<td>2</td>
<td>285.5</td>
<td>285.9</td>
<td>-0.14</td>
<td>99.7</td>
</tr>
<tr>
<td>3</td>
<td>538.2</td>
<td>537.5</td>
<td>0.14</td>
<td>98.8</td>
</tr>
<tr>
<td>4</td>
<td>910.3</td>
<td>910.4</td>
<td>-0.07</td>
<td>97.9</td>
</tr>
<tr>
<td>5</td>
<td>1100.4</td>
<td>996.9</td>
<td>10.38</td>
<td>95.8</td>
</tr>
<tr>
<td>6</td>
<td>1347.5</td>
<td>1334.4</td>
<td>0.98</td>
<td>95.3</td>
</tr>
</tbody>
</table>
6.7.2 Mode Shapes and Frequencies as Responses

Figure 6.12 (Sensitivities-mode shapes as responses-10mm crack)

Figure 6.13 (Parameter Modifications-mode shapes as responses-10mm crack)
Figure 6.14 (3D Sensitivity Matrix-frequency/mode shape responses-10mm Crack)

Table 6.7 (Updated results for beam-mode shapes responses-10mm Crack)

<table>
<thead>
<tr>
<th>Mode Shape Pairs</th>
<th>FEA(Hz)</th>
<th>EMA(Hz)</th>
<th>Diff(%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.3</td>
<td>95.3</td>
<td>-1.12</td>
<td>99.6</td>
</tr>
<tr>
<td>2</td>
<td>288.2</td>
<td>285.9</td>
<td>0.78</td>
<td>99.8</td>
</tr>
<tr>
<td>3</td>
<td>534.3</td>
<td>537.5</td>
<td>-0.6</td>
<td>99.2</td>
</tr>
<tr>
<td>4</td>
<td>917.9</td>
<td>910.9</td>
<td>0.77</td>
<td>98.9</td>
</tr>
<tr>
<td>5</td>
<td>1080.4</td>
<td>996.9</td>
<td>8.37</td>
<td>95.8</td>
</tr>
<tr>
<td>6</td>
<td>1361.8</td>
<td>1334.4</td>
<td>2.06</td>
<td>95.1</td>
</tr>
</tbody>
</table>
6.8 Double 10mm Experimental Crack Updating Results

6.8.1 Frequencies only as Responses

Figure 6.15 (Sensitivities-frequencies as responses-Two 10mm cracks)

Figure 6.16 (Parameter Modifications-frequencies as responses-Two 10mm cracks)
6.8.2 Mode Shapes and Frequencies as Responses

<table>
<thead>
<tr>
<th>Mode Shape Pairs</th>
<th>FEA(Hz)</th>
<th>EMA(Hz)</th>
<th>Diff(%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>95.3</td>
<td>95.3</td>
<td>-0.01</td>
<td>99.8</td>
</tr>
<tr>
<td>2</td>
<td>275.9</td>
<td>276.6</td>
<td>-0.24</td>
<td>98.7</td>
</tr>
<tr>
<td>3</td>
<td>510.48</td>
<td>509.4</td>
<td>0.22</td>
<td>97.5</td>
</tr>
<tr>
<td>4</td>
<td>864.1</td>
<td>864.1</td>
<td>0.00</td>
<td>97</td>
</tr>
<tr>
<td>5</td>
<td>1090</td>
<td>992.2</td>
<td>9.85</td>
<td>95.3</td>
</tr>
<tr>
<td>6</td>
<td>1299.6</td>
<td>1317.2</td>
<td>-1.34</td>
<td>93.3</td>
</tr>
</tbody>
</table>

Figure 6.17 (Sensitivities-mode shapes as responses-Two 10mm cracks)
Figure 6.18 (Parameter Modifications-mode shapes as responses-Two 10mm cracks)

Figure 6.19 (3D Sensitivity Matrix-frequency/mode shape responses-Two 10mm Cracks)
Table 6.9 (Updated results for beam-mode shape responses-Two 10mm Cracks)

<table>
<thead>
<tr>
<th>Mode Shape Pairs</th>
<th>FEA(Hz)</th>
<th>EMA(Hz)</th>
<th>Diff(%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>95.6</td>
<td>95.3</td>
<td>0.31</td>
<td>99.8</td>
</tr>
<tr>
<td>2</td>
<td>267.8</td>
<td>276.6</td>
<td>-3.15</td>
<td>99.5</td>
</tr>
<tr>
<td>3</td>
<td>520</td>
<td>509</td>
<td>2.08</td>
<td>98.6</td>
</tr>
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<td>4</td>
<td>870.9</td>
<td>864</td>
<td>0.79</td>
<td>98.1</td>
</tr>
<tr>
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<td>1090</td>
<td>992.2</td>
<td>9.86</td>
<td>95.7</td>
</tr>
<tr>
<td>6</td>
<td>1417.9</td>
<td>1317.2</td>
<td>7.65</td>
<td>92</td>
</tr>
</tbody>
</table>

**6.9 Discussions of Results**

The results presented in section 6.5-6.8 are discussed along with the relevance of all the graphs and tables listed. There are four sets of results which are presented being, parameter sensitivity, parameter changes, frequency and MAC comparisons and the 3D sensitivity matrix.

Model quality is the first results which are shown, with the use of the parameter sensitivities to show the quality of the model. There are two things which must be checked for model quality with sensitivity based model updating procedures. The first being the convergence of the results, this is quite well developed within the software and is highlighted to the user if convergence is not found. All the results presented have convergence of the results, but graphs of the parameter convergence was omitted for report brevity, but the reader is directed to the supplementary CD provided to see these results, and in fact all other results in the Fem-Tools projects for all model updating. The second result which must be checked within sensitivity based
model updating procedures for model quality assurance is the distribution of the sensitivities compared to the distribution of the parameter changes. As the parameter changes within the model are weighted by the sensitivity matrix then if the distribution of the sensitivity and change in parameters is similar in shape then the results generally should not be relied upon. This is one fundamental limitation to sensitivity based model updating procedures and is often not discussed by the proprietary software itself [1][32]. These sensitivities are shown along the length of the beam for the choice of parameters. The 3D matrix is presented to show the weighting of each response towards the parameter changes.

The parameter changes along the elements are shown as a method of detecting the location and presence of a local fault with the greatest parameter changes highlighting its location. The location of the first simulated crack at 540mm from the datum end is located in the 11th and 19th element within the finite element model and should be kept in mind when analysing the results shown. The location of the second simulated crack at 190mm from the datum end is located in the 4th and 24th element in the finite element model.

Finally the comparison of the updated frequencies and the MAC for each mode is shown to have a quantitative measure of how close the updated model resembles the measured modal parameters.

6.9.1 FEA simulated Crack

A single 5mm deep crack was modelled in FEA at the same location of the first introduced crack for the experimental results.

The parameter changes for the frequency and mode shapes as responses are only shown for the FEA simulation so that the reader can see the predicted updating parameter shapes that should be seen in the experimental results without the
corruption of measurement noise. Figure 6.2 and Figure 6.3 show the parameter modifications for the frequency and mode shapes as responses respectively with the location of the crack also highlighted with a curser. It can be seen that both methods accurately locate the single 5mm crack. Figure 6.4 is a contour plot of the parameter changes over the length of the beam to give the reader a better understanding of the visual changes which take place in the FEA model.

6.9.2 5mm Experimental Crack

Frequencies only as responses are first presented with then the results of the mode shapes and frequencies as responses presented. The sensitivity of the parameters is shown, with Figure 6.5 and Figure 6.7 showing the sensitivities across the length of the beam, it can be seen that these are different from the parameter changes in Figure 6.6 and Figure 6.8 across the same elements of the beam so that the models quality can be assumed to be valid.

Figure 6.6 and Figure 6.8 show the parameter changes across the length of the beam for the frequency only, and mode shapes and frequencies as responses respectively. The greatest parameter changes for the frequency only responses happening at elements 10 and 11, which highlights the smearing of the parameter changes across the elements on either side of the axis of symmetry, plotted also on all the parameter change results is a curser at the location of the crack. The change in parameters at the point of greatest change is approximately a 20% modification. For the frequency and mode shapes as responses, the greatest parameter changes happen at element 11 but the largest change between 11th and 12th elements is what gives away to the presence and location of the crack, even though it is slightly displaced from the actual location of the crack. The greatest change in parameters for this
Chapter 6

response case is only around 10% but at the location of the large change in parameters from adjacent elements with a change of approximately 20% occurring.

The 3D sensitivity matrix shown in Figure 6.9 shows the amount of weighting each response has for each parameter. The responses of 1-4 are the first four bending modes, then the rest of the responses show the responses for the mode shape measurements. It can be seen that the sensitivity of the mode shapes measurements is quite high and higher than the sensitivity for the frequencies as responses for most parameters, this shows that the introduction of the mode shapes as responses is both valid and desirable.

Finally the comparison of the updated frequencies verses the experimentally measured frequencies is shown in Table 6.4, as well as the MAC for each mode shape. It can be seen that the difference of frequencies is very good with a value in the order of 0.5% and less for frequencies only as responses than for mode shapes and frequencies as responses shown in Table 6.7. There is a slight increase in the difference when the mode shapes are added as responses, most likely due to the fact now that the mode shapes with 100% weighting, are now included into the penalty function which needs to be reduced in the updating algorithm. The MAC values for when the mode shapes are also included as responses shows better values, obviously as the MAC is only based on the mode shape correlation as seen in equation [6.1], and with the mode shapes being introduced and their responses updated then the MAC would increase. This is also a good indication that the inclusion of the mode shapes has had a positive impact on the whole finite element model more closely resembling the measured results.
6.9.3 10mm Experimental Crack

Frequencies only as responses are first presented with then the results of the mode shapes and frequencies as responses presented. The sensitivity of the parameters is shown, with the Figure 6.10 and Figure 6.12 showing the sensitivities across the length of the beam, it can be seen that these are different from the parameter changes in Figure 6.11 and Figure 6.13 across the same elements of the beam so that the models quality can be assumed to be valid.

Figure 6.11 and Figure 6.13 show the parameter changes across the length of the beam for the frequencies only as responses, and mode shapes and frequencies as responses, respectively. The greatest parameter changes for the frequency only responses happening at elements 10 and 11, which highlights the smearing of the parameter changes across the elements on either side of the axis of symmetry, plotted also on all the parameter change results is a cursor at the location of the crack. The change in parameters at the point of greatest change is approximately 40% modification. For the frequency and mode shapes as responses, the greatest parameter changes also happen at element 11, but without the smearing of the updated parameters with a sharp peak in the parameter changes seen in Figure 6.13. The greatest change in parameters for this response case in approximately 40% as was seen in the updating used in the frequency only response case.

The 3D sensitivity matrix shown in Figure 6.14 shows the amount of weighting each response has for each parameter. The responses of 1-4 are the first four bending modes, then the rest of the responses show the responses for the mode shape measurements. It can be seen that the sensitivity of the mode shapes measurements is quite high and but are lower in general than the sensitivities to the frequencies for most parameters, the introduction of the mode shapes as responses
therefore can be seen as advantageous to the updating algorithm, but it is still more dominated by the frequency responses.

Finally the comparison of the updated frequencies verses the experimentally measured frequencies are shown in Table 6.6, as well as the MAC for each mode shape. It is shown that the difference of frequencies is very good with a value in the order of 0.5% and less for both frequencies only as responses and mode shapes and frequencies as responses later shown in Table 6.7. There is a slight increase in the difference when the mode shapes are added as responses, most likely due to the fact now that the mode shapes with 100% weighting, are included into the penalty function which needs to be reduced in the updating algorithm. The MAC values for the when the mode shapes are also included as responses shows better values, obviously as the MAC is only based on the mode shape correlation as seen in equation [6.1], and with the mode shapes being introduced and there responses updated then the MAC would increase. This is also a good indication that the inclusion of the mode shapes has had a positive impact on the whole finite element model more closely resembling the measured results.

6.9.4 Double 10mm Experimental Crack

Frequencies only as responses are once again first presented with then the results of the mode shapes and frequencies as responses presented. The sensitivity of the parameters is firstly shown, with the Figure 6.15 and Figure 6.17 showing the sensitivities across the length of the beam, it can be seen that these are different from the parameter changes in Figure 6.16 and Figure 6.18 across the same elements of the beam so that the models quality can be assumed to be valid.

Figure 6.16 and Figure 6.18 show the parameter changes across the length of the beam for the frequency only, and mode shapes and frequencies as responses,
respectively. The greatest parameter changes is for the frequency only as responses happening at elements 10 and 11 and then at element 4 and 17. This case shows the greatest indicator of how the parameters are smeared across the elements around the axis of symmetry. The same parameter changes for elements on both sides of the axis of symmetry show how the damage location could not be totally located, plotted also on all the parameter change results is a cursor at the location of the crack. The change in parameters at the point of greatest change is approximately 100% modification at elements 10 and 11, and 70% modification at elements 4 and 17. The parameter changes of 100% and 70% for the elements along the beam show that the use of the parameter change magnitude can not be directly related to the depth of the crack as both cracks are of equal depth and a 100% parameter modification is not physically possible. For the frequency and mode shapes as responses, the greatest parameter changes also happen at the equivalent element 12 and 18, but without the smearing or doubling of the peaks around the axis of symmetry of the updated parameters with the sharp peak in the parameter changes seen in Figure 6.18. The greatest change in parameters for this response case is approximately 60% and 100% at the location of element 12 and 18 respectively.

The 3D sensitivity matrix shown in Figure 6.19 shows the amount of weighting each response has for each parameter. The responses of 1-4 are the first four bending modes, then the rest of the responses show the responses for the mode shape measurements. It can be seen that the sensitivity of the mode shape measurements is quite high and again begin to dominate the response sensitivities over the frequency responses, the introduction of the mode shapes as responses therefore can be seen as advantageous to the updating algorithm.
Finally the comparison of the updated frequencies verses the experimentally measured frequencies are shown in Table 6.8, as well as the MAC for each mode shape. It is shown that the difference of frequencies is very good with a value of in the order of 1% and less for frequencies only as responses. When the mode shapes are then used as responses the difference between the frequencies is starting to become much greater with an average value approximately 2-3% for most modes this is shown in Table 6.9. There is a slight increase in the difference when the mode shapes are added as responses, most likely due to the fact now that the mode shapes with 100% weighting, are included into the penalty function which needs to be reduced in the updating algorithm. The MAC values for the when the mode shapes are also included as responses shows better values, obviously as the MAC is only based on the mode shape correlation as seen in equation [6.1], and with the mode shapes being introduced and their responses updated then the MAC would increase. This is also a good indication that the inclusion of the mode shapes has had a positive impact on the whole finite element model more closely resembling the measured results.

6.10 Conclusion of Updating Results

All the results show the presence and location of the simulated cracks with an increase in parameter changes between the 5mm deep single crack and the 10mm deep single crack to show that an increase in damage has taken place. The change in parameters is basically proportional to the amount of damage through the cross section for the results for the single simulated crack but when the double cracks are to be detected the updated parameters are much less able to give the relative severity of damage. This is most likely due to the fact that this is a sensitivity based model updating method and therefore damage of the same severity but in different locations with greater or lesser sensitivities will show different levels of parameter changes.
The later results with the double simulated crack causes parameter changes which are not physically viable, much discussion by other authors has been undertaken on the physical realisation of parameter changes and what limits should be set so that they can not change to non physical values, which don’t physically model the reality of the condition[1][31], however as only detection of damage is sort in this updating instance the physical realization of the parameters is not of paramount importance.

All the results with the use of just the frequencies as responses, show a very distinct updating of parameters at the location of the simulated crack however with the phenomenon of smearing the updated parameters across the adjacent elements across the axis of symmetry. When the mode shapes are also included as response parameters in the updating algorithm the presence of the crack is also observable, without the smearing of the parameter changes across the axis of symmetry, however the crack is highlighted by the greatest change in adjacent elements rather than the greatest parameter change in an element itself. The results with mode shapes included as responses however shows a greater deal of noise, or parameter changes in elements which are undamaged in the real case, compared to the very steep spike in parameter changes with the frequencies only as responses. This brings to the conclusion that the use of frequencies only as responses would be preferred as the method of detecting damage presence and location within a structure. If an axis of symmetry exists and the damage needs to be located as to which side it lays on, then an introduction of responses into the updating algorithm which hold spatial information would be needed, such as mode shapes.

The massive reduction in the amount of measurements and processing of the data if only frequencies are used as updating responses also means that the use of only having to take measurements at one location and no curve fitting of the FRFs and
mode shape construction needs to be done, can only lead to the conclusion, along with the superior ability to highlight the position and severity of the crack compared to when the mode shapes are included as responses, means that frequencies only as responses would have to be preferred.
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7 Mode Shape Curvature Analysis

The sensitivity of mode shapes to the introduction of damage into a system has long been known to be greater than the sensitivity of frequency changes due to damage [1][16][17][20]. The introduction of a simulated crack into a system will cause a discrete change in cross section at the location of the crack. It is already clear that the mode shape will change and as the mode shapes curvature is the second derivative of the mode shape the curvature will also change due to the introduction of the crack. However the discrete change in cross section at the location of the crack and observing the bending moment curvature equation, \( M_{\text{moment}} = EI \kappa \), shows that a discrete change in cross section will cause a discrete change in curvature.

From the measurement and construction of the mode shape vectors a curve must be fitted such that the second derivative can be taken and the mode shapes curvature can be taken. The method for the curve fitting and subsequent derivation of the mode shape curvature will first be presented. The choice of mode shapes used for the analysis of the results is then undertaken.

Presentation of the results obtained from the FEA-simulation and the experimental analysis, followed by a discussion and conclusion of all the results are then undertaken.

7.1 Curve Fitting Measurement Data

Firstly an attempt to fit a curve to the experimental mode shapes such that the second derivative is continuous across the beam length but can also highlight a discrete change in curvature can be observed at the location of the simulated crack. Observance of the analytical mode shape curvatures constructed from derivation of the Euler-Bernoulli theory shown in Figure 3.3 should first be undertaken and an
appropriate choice of curve fitting method, must at the very least show a shape similar to the curvature shape shown.

Fitting of a Lagrange polynomial to the mode shape data so as to help smooth out the noise in the mode shape measurements has been suggested and undertaken by many researchers[14][27]. However, the use of a Lagrange polynomial fitted to the first bending mode shape generated using finite element analysis, and its curvature is seen in Figure 7.1, it can clear that the curvature is not in the same form as the analytical curvature shown in Figure 3.3, and therefore questions the validity of this method. By the very fundamental definition of the fitting of an entire polynomial to a data set, then the second derivative will not have a discrete change at the point of introduced damage, and will only be effective at showing the presence of damage over a large area across the structure.

![First Bending Mode and Curvature](image)

Figure 7.1 (First Mode shape/Curvature with Lagrange fitted curve-nocrack)
Now that it has been explained that a global polynomial fitting of the data will not be a sufficient method for fitting the data, then it is clear that a piece wise polynomial fit would enable a polynomial description of the experimental mode shape data but also pass through all the measurement points such that a discrete change in curvature can be observed. A few methods of fitting piece wise polynomials to data sets are available, the most widely used functions are the fitting of cubic splines or hermite cubic polynomials. Cubic splines have the property that at all measurement points are passed through and derivatives of the curve are continuous, where as the hermite cubic polynomial is only continuous at all points in its first derivative, but also passes through all measurement pints. I would then seem that the choice of cubic spline fitting would be the logical choice however, when only a cubic spline is used to fit the data from the first bending mode of a finite element, discrete changes still occur in the undamaged case as seen in Figure 7.2 due to the oscillatory nature of the cubic spline fitting of the mode shape. The shape of the curvature in Figure 7.2 shows a shape similar to the analytically derived curvatures seen in Figure 3.3 and shows the much greater ability of the cubic spline fit over a global polynomial fitting. To over come these oscillations of the cubic splines through the fitting of the mode shapes such that the curvature will be smooth, can be done by increasing the measurement sampling, so that the difference between underlying function of the real curvature and the cubic spline interpolation is minimal. Another way of smoothing the curvature with the cubic spline fitting is to stop the oscillations of the splines. This is where the hermite polynomial fitting is advantageous.
Fitting of the mode shapes just using a hermite cubic polynomial, as would be expected causes a very discretized curvature due to the second derivative is not held continuous by definition of the hermite piece wise cubic polynomial data fitting as seen in Figure 7.3.

The ability of the hermite piece wise polynomial fitting to reduce the oscillations which are present in the fitting with cubic splines can be seen in Figure 7.4 with the flatter interpolation between data points produced from the hermite piecewise polynomial and the oscillatory nature of spline fitting of data points is also seen [5]. It is clear that if the direction of the spline fitting changes in its direction between points due to these oscillations seen in Figure 7.4, then spurious discrete changes in curvature would occur as Figure 7.2 shows.

The exploitation of both the desirable properties of both piece wise curve fitting types is then used. This is done by first fitting the data with a hermite
polynomial and then sampling this fitted data at three times the original sample point rate and then fitted with a cubic spline polynomial. This means that the final function is continuous in the second derivative but also the oscillations which are often present in the cubic spline fitting have been reduced.

Figure 7.5 shows the results of the first bending mode constructed with finite elements and the derived curvature. It is clear that this curve shows the shape of the analytical curvatures derived from the Euler-Bernoulli equation as well as being smooth and without discrete changes due to the curve fitting errors. This is the form which will be used in fitting all the mode shapes with damage introduced and assessed for the change in curvature. The curvatures are also fixed at zero at the end points to make sure the boundary conditions are held. The analytical second derivative is then derived using the matlab function derivative command of the fitted cubic splines. This method of hermite and spline piece wise fitting of the curvatures as shown a smoother curvature than has been found in any of the literature reviewed. Pandey et al used a central differences method to calculate the second derivative of mode shapes and for finite element analytical results showed much more discretized curvatures for the undamaged case. The matlab script used for the fitting of the data, with the variables changed for each damage case can be seen in Appendix B.
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Figure 7.3 (First Mode shape/Curvature with Hermite fitted curve-nocrack)

Figure 7.4 (Comparison of Spline and Hermite Curve Fitting Properties) [5]
7.2 Choice of Mode Shapes for Analysis

The first four bending modes were only considered for analysis with this method of assessing the change in modal parameters for the same reasons which have been previously discussed about the measurements quality for the higher order modes. The first four bending modes for the beam without a simulated crack are shown in Figure 7.6. It is seen that the bending modes are very well defined with only a small deviation in the fourth bending mode, however the measurement of the fourth bending mode degrades with the later measurements and it is therefore was not used in this method of analysis, as with the mode shape curvatures being directly derived from the mode shapes, the measurement quality is paramount.

The lower mode shapes are also chosen to be used for the fact that when the curvature differences are taken from the curvature with and without damage, the points of zero curvature, such as the inflection points and end points of the curves,
where the curvature approaches zero, so that when the difference of very small numbers are taken then there is a lot of domination by measurement noise, and therefore higher order modes have more areas of noise dominated areas than the lower order curvature modes [16].

![Fist Four Bending Modes](image)

**Figure 7.6 (Bending Modes-without Crack)**
7.3 FEA simulation of Crack, Curvature Analysis

![Graph showing first bending mode curvature with 5mm crack at center](image)

Figure 7.7 (First Mode Shape Curvature-with 5mm FEA simulated Crack)

7.4 5mm Experimental Crack, Curvature Analysis
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Figure 7.8 (Bending Modes-with 5mm Crack)

Figure 7.9 (First Mode Shape Curvature-without Crack)
First bending mode curvature with 5mm crack

![First Bending Mode Curvature with 5mm Crack](image)

Figure 7.10 (First Mode Shape Curvature-5mm Crack)

Curvature Difference First Bending Mode

![Curvature Difference First Bending Mode](image)

Figure 7.11 (First Mode Shape Curvature Difference- 5mm Crack)
Figure 7.12 (Second Mode Shape Curvature Difference- 5mm Crack)

Figure 7.13 (Third Mode Shape Curvature Difference- 5mm Crack)
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7.5 10mm Experimental Crack, Curvature Analysis

Figure 7.14 (CDF with 5mm Crack)

Figure 7.15 (Bending Modes-with 10mm Crack)
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**Figure 7.16 (First Mode Shape Curvature-10mm Crack)**

**Figure 7.17 (First Mode Shape Curvature Difference- 10mm Crack)**
Figure 7.18 (Second Mode Shape Curvature Difference- 10mm Crack)

Figure 7.19 (Third Mode Shape Curvature Difference- 10mm Crack)
7.6 Double 10mm Experimental Crack, Curvature Analysis

Figure 7.20 (CDF with 10mm Crack)

Figure 7.21 (Bending Modes-with Two 10mm Cracks)
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Figure 7.22 (First Mode Shape Curvature-Two 10mm Cracks)

Figure 7.23 (First Mode Shape Curvature Difference-Two 10mm Cracks)
Figure 7.24 (Second Mode Shape Curvature Difference-Two 10mm Cracks)

Figure 7.25 (Third Mode Shape Curvature Difference-Two 10mm Cracks)
7.7 Discussions of Results

For all sets of results the four bending modes, curvature for the first bending mode, curvature difference for all modes and the Curvature Damage Factor, CDF which was formulated by Abdel et al and is defined by,

\[
CDF = \frac{1}{N} \sum_{n=1}^{N} |\nu''_n - \nu'^n| \tag{7.1}
\]

where \( \nu''_n \) and \( \nu'^n \) are the curvatures without and including damage respectively.

The bending modes are first presented for each damage case to show the quality of the mode shape measurements that the derivation of the curvatures will then be based upon so the reader can help assess the quality of the data later presented.

Measuring the curvature directly from the damaged structure without the need of measurement of the structure before damage is advantageous as access or
knowledge of the structure before the structure has undergone damage is often not available for real structures. Therefore the curvature for the first bending mode measured directly from the measurements is presented for each damage case without comparison to the curvatures found before the damage was introduced.

The curvature for the first bending mode shows the amount of discrete changes and errors in the curvature measurement without the simulated crack within the beam as seen in Figure 7.9. This shows the need to compare the mode shape curvatures with damage in the structure relative to the mode shape curvatures without damage and leads to the reasoning behind the presentation of the mode shape curvature differences for all the first three bending mode shapes for all the damage cases. The reason for all the mode shape curvatures shown for each damage case is to highlight that the location of the simulated cracks has on the effect on the change in curvature for different modes, such as the second bending mode not being particularly effected by the first just off centre crack.

Finally a method of combining all the measured mode shape curvature differences across all measured modes is presented by the CDF. This CDF is the average of the curvatures across all modes and provides a method of assessing the location of damage across many modes, where it might not be present within just one mode shape, in just one function variable.

The locations of the introduced simulated cracks are highlighted in all graphs by a red cursor line.

7.7.1 FEA Simulated Crack

The simulation of a 5mm crack in a FEA model was produced at the centre of the beam to show the ability of assessing the change in modal curvatures to detect the presence and location of a crack without the presence of noise as is in experimental
data. The change in curvature can be seen in Figure 7.7 with a large discrete change in curvature at the point of the crack as was predicted. The general curvature shape can be compared to that seen in Figure 7.5 and it is seen that they are of the same shape apart from at the location of the crack.

### 7.7.2 5mm Experimental Crack

The first bending mode shape curvature in is firstly shown Figure 7.10 measured directly from the mode shape of the first damage case. This shows a large peak at the location of the simulated crack with a value 50% higher than the highest peak from the noise in the signal. The difference in curvatures for bending modes one through to three can be seen in Figure 7.11-Figure 7.13 respectively. Using the relative difference between the first bending mode curvature with and without the introduced crack has increased the peak produced by the discrete change in curvature to a value of more than twice the values of the highest peak from the noise in the curvature shape. The location of the crack is not the dominant peak in the second bending mode curvature difference as would be expected however in the curvature difference for the third bending mode it is once again the most dominant peak in the curvature. It is clearly evident on the location and presence of the crack from the observance of these graphs.

Finally the Curvature Difference Function, CDF, is plotted with a large peak over twice the height of the most dominant peak from noise in the curvature seen in Figure 7.14. It is also worth noting that the biggest point of measurement noise is at the locations near the end of the curvature where the measurement of the end conditions where often not met, and had to be enforced when fitting the cubic spline functions.
7.7.3 10mm Experimental Crack

The first bending mode shape curvature is firstly shown in Figure 7.15 measured directly from the mode shape of the second damage case of a single 10mm crack. This shows a large peak at the location of the simulated crack with a value twice as high as the highest peak from the noise in the signal. The difference in curvatures for bending modes one through to three can be seen in Figure 7.17-Figure 7.19 respectively. Using the relative difference between the first bending mode curvature with and without the introduced crack has increased the peak produced by the discrete change in curvature to a value of more than three times the values of the highest peak from the noise in the curvature shape and a value almost twice the height of the peak for the 5mm crack. The location of the crack is the dominant peak in the second bending mode curvature difference but is only slightly higher than the highest peak from noise. The curvature difference for the third bending mode is once again the most dominant peak in the curvature but with a value only approximately twice that of the highest noise peak. It is clearly evident on the location and presence of the crack from the observance of all these graphs.

Finally the Curvature Difference Function, CDF, is plotted with a large peak over four times the height of the most dominant peak from noise in the curvature seen in Figure 7.14. This value is almost twice the height of the CDF peak for the 5mm crack and indicates more severe damage.

7.7.4 Double 10mm Experimental Crack

The first bending mode shape curvature is firstly shown in Figure 7.22 measured directly from the mode shape of the third damage case of double 10mm cracks. This shows a large peak at the location of the first simulated crack with a
value over twice as high as the highest peak from the noise in the signal, the second crack location is present but is certainly not dominant. The difference in curvatures for bending modes one through to three can be seen in Figure 7.23-Figure 7.25 respectively. Using the relative difference between the first bending mode curvature with and without the introduced crack has increased the peak produced by the discrete change in curvature for the first 10mm crack just off centre to a value of more than three times the values of the highest peak from the noise in the curvature shape and approximately the same order above the noise as for the single 10mm crack, as would be expected. The location of the second crack is not evident in the difference curvature for the first bending mode The location of the second 10mm crack is the dominant peak in the second bending mode curvature difference with a value around 50% higher than the highest noise peak, the first 10mm crack however is not evident. The curvature difference for the third bending mode shows the location of the second simulated crack as the most dominant peak over three times the height of the highest noise peak and the first 10mm is the next dominant peak over 50% higher than the biggest noise peak. The combination of all these graphs is able to highlight the location of both cracks, but the use of just one mode shape might miss the location of one of the cracks, this really brings the need for a function which combines a series of mode shapes in one value, such as the CDF.

Finally the Curvature Difference Function, CDF, is plotted with a large peak over twice the height of the most dominant peak from noise in the curvature seen in Figure 7.14 and also a large change in curvature at the location of the second 10mm crack.
7.8 Conclusion of Curvature Analysis Results

The presence and location of the crack is evident in all the damage conditions with the single 5mm and 10mm crack as well as the double 10mm crack. The analysis of the direct curvature measured for the first bending mode in each case shows the presence of the simulated located just off centre, and it is also noted that the shape of the mode shape is in fact almost not visibly changed due to the presence of the crack so it highlights the ability of the mode shape curvature to increase the sensitivity to the presence of damage. The increase in the relative curvature for the 5mm and 10mm crack was almost double giving a measure of the severity of the damage as well as its location. The difference in curvature relative to the undamaged case and the damaged cased for the first through to third bending mode for each damage case showed how each mode is effected differently by the different locations of the crack and that for the presence of damage at all locations within the beam, a combination of mode shapes needs to be analysed. This was done by using the average of curvature difference for each mode using the CDF. The CDF was a single functional variable which was able to both locate the cracks at different locations and relative severity to quite good accuracy. The increase in the CDF value was basically twice as large for the 10mm crack as it was for the 5mm crack and also equal for the double crack results.

All the results presented show that analysis of the change in mode shape curvature due to the introduction of a local fault is a valid, method for detection, location and diagnosis of a crack within a structure.
8 Discussions and Conclusions

The evaluation and discussion of the differences and strengths of the techniques used to assess the change in modal parameters such that the presence and location of the introduced crack could be detected is to be presented within this chapter.

Conclusions and summary of the entire project overview and the work completed will also feature along with suggestions for future work into a similar project.

8.1 Comparisons of Methods

The comparisons of the two methods for assessing the change in modal parameters to detect the presence and location of the simulated local fault, needs to first determine each methods ability to detect the presence of the simulated fault without prior knowledge of its location or presence. The results and discussions in chapter 6 and 7 clearly show that the presence and location of the initial 5mm single crack is able to be located and presents a change in the assessed parameters great enough that it is more than 50% greater than the changes in the non-damaged sections which are assumed attributed to noise. The increase in the crack depth to 10mm creates the changes in the measured parameter for each method used and therefore shows that there is a sense of general severity present in each method. The model updating method used with mode shapes as responses and the modal curvature method were able to discretely locate the cracks location however the model updating method with frequencies only as responses could only find the cracks location at two points across the axis of symmetry. There was not however any specific numerical value that could be stipulated for either method that could be described for.
any structure which differs from the one studied that could be used to detect the presence or location of local damage. This is shown in the results by the variation in the parameters which showed the damage location in the structure for the 10mm crack with the single 10mm crack and then when the second 10mm crack was introduced. Therefore it is deduced that the absolute change in parameters could not be transferred to a different structure to give a general numerical value which would show the presence and location of damage within the structure.

With the detection and location of the faults able to be found in the structure using both methods, then further evaluation criterion must be introduced to assess the valid use of either method. Two more criteria for evaluation will be used, the robustness of a method to withstand measurement noise and the amount of manipulation or measurement time required to obtain the results. The use of mode shapes in the model updating technique as well as the mode shape curvature assessment method, means obviously the mode shapes need to be measured. This requires a large number of measurements to be taken across a grid of measurement points across the structure, with increasing number of measurement points for complex structures, to define the shape. Curve fitting methods are then needed to construct the mode shapes from the frequency response measurement across the structure. The model updating procedure using only the frequencies as responses requires in theory only one measurement point to be taken so it is very much less measurement intensive, however for the model updating technique a adequate finite element model also needs to be produced for the structure.

The computational calculations that are needed for the model updating method are much more intensive than that required for the modal curvature method after the construction of the mode shapes, as a simple routine is able to be developed to
construct the curvatures as shown in Appendix B, however the calculations for the model updating method are able to be automated with the available proprietary software. If more complex structures are to be analysed then the time the computations take might then become an issue.

The choice of which method is more relevant to use for the detection of faults within a structure is highly dependant on the available software, and the structures ability to be modelled accurately using finite element analysis, but the frequency only response based model updating procedure would have to be the most robust method which is least effected by measurement noise, and is very simple to measure and requires very little manipulation of the data after measurement. The location of the crack is limited however by symmetry of the structure and can only be found to within a location of twice as many points as the number of axis of symmetry it lays around. As long as this inability to locate the exact position of the crack due to the symmetry of a structure this method seems the best and most reliable option which was explored, although both methods were able to show their ability to detect and locate the simulated crack.

\section*{8.2 Conclusions and Summary}

Two independent methods for assessing the change in modal parameters of a structure in order to find the presence and location of a simulated crack in a mild steel rectangular section beam were compared. Experimental modal analysis was conducted on the beam with the natural frequencies and mode shapes found for the beam, along with a finite element model constructed for the structure.

The first method evaluated for its ability to detect the local fault was the model updating method which allows the parameters of a finite element model of the structure to vary such that the difference in responses for the finite element model and
experimentally measured responses are minimised. The detection of the introduced simulate cracks, were detected using this method with using just the natural frequencies and then also including the mode shapes as responses. The natural frequencies as responses seemed more robust in the presence of noise but were unable to directly locate the position of the fault due to the smearing of the parameter changes across the axis of symmetry, were as with the mode shapes as response were able to directly locate simulated crack. When using just the natural frequencies and with the inclusion of the mode shapes as responses also saw a greater change in parameters when the crack depth was increased so that this method therefore was able to detect the severity of a fault also.

The second method evaluated was the change in the modal curvatures due to the introduction of the simulated crack, which will cause a discrete change in curvature at the point of damage location. The results showed that the detection of both introduced cracks was able to be done if the analysis of the right modes were undertaken, and with the use of the Curvature Difference Function. Increase in the change in modal curvature was also apparent with the increase in depth of the first introduced crack from 5mm to 10mm, which identified the method as being able to also adequately assess the severity of damage.

Comparisons of the two methods on the evaluation of their ability to detect and locate the crack, stability in the presence of noise and computation time was used to decide on the best method to use for damage detection. It is concluded that the model updating method with the use of only frequencies as responses is the best method for crack detection, due to its low amount of measurements needed and robustness in the presence of noise, but only if the smearing of the detection point across the axis of symmetry is not an issue in the detection.
8.3 Suggestions Future Work

There exits a large number of possibilities for future work on the detection of faults due to the changes in a structures modal parameters. Two main directions could be taken for future work, building on the work and conclusions presented by the two methods assessed within this thesis, or comparing other methods which were discussed in the literature review.

If further work was to be considered on building on the conclusions that were presented within, the use of the natural frequencies as updating parameters for detection of a crack within a more complex structure, with differing boundary conditions would have a great deal of scope for work and development.

Taking a different line of theory to obtain the detection of a local fault due to the change in modal parameters could be done by the wide range of other methods that were previously presented. Four different methods of assessing the change in modal parameters presented hold the greatest promise for further study. The direct use of the Frequency Response measurements in the modal model updating method instead of the use of mode shapes as responses and there ability to have better convergence of parameters and robustness in the presence of noise to be assessed [1][30] would be a very good extension of this thesis. The refinement of the mesh density as a parameter with the updating of parameters in the modal model updating algorithm would also present good scope for further development and study, however would require a fair degree of interfacing with a pure finite element analysis program with the updating program such that a routine for the updating of the FEA mesh is updated within the area of greatest parameter changes [33].

Comparisons of the methods presented within this thesis or to the other methods mentioned above could also be done to the analytical model derived.
Christides et al [19] of a Euler-Bernoulli model for the mode shape deflections for a symmetrical crack. The most interesting method for assessing the change in the modal parameters due to the introduction of a local fault could be done by the direct measurement of the dynamic strain energy the structure under excitation to assess the change in modal curvature due to the local fault, without the increased measurement noise that is taken from derivation of the curvature from the mode shape measurements [13]. The ability for the use of this last method however would be very dependant on the available equipment for the measurements.

### 8.4 Concluding Statement

Detection of local faults in structures is still a subject of ongoing research with no one method taking the forefront in engineering structural damage assessment, however a wide variety of methods exits which when used under the right circumstances are very accurate in local fault detection in structures.
References


Appendix A. Derivation of motion Equation for continuous beam.

Free Vibration case:

Figure A.1 (Forces on a transverse element of a beam $dx$)

\[\Sigma F_x = 0\]

\[V(x,t) - \rho A(x) dx \frac{\partial^2 u}{\partial t^2}(x,t) - dV(x,t) - V(x,t) = 0\]

\[\rho A(x) dx \frac{\partial^2 u}{\partial t^2}(x,t) = -dV(x,t)\] \[\text{[A.1]}\]

\[\Sigma M_o = 0\]

\[M(x,t) + dM(x,t) - M(x,t) - \frac{dx}{2} V(x,t) - \frac{dx}{2} dV(x,t) = 0\]

\[dM(x,t) - dx[V(x,t) + \frac{dV}{2}(x,t)] = 0\] \[\text{[A.2]}\]
As \( dV = \frac{\partial V}{\partial x} \, dx \) and \( dM = \frac{\partial M}{\partial x} \, dx \) then

\[ [A.1] \Rightarrow \frac{\partial V}{\partial x} (x,t) + \rho A(x) \frac{\partial^2 u}{\partial x^2} (x,t) = 0 \] [A.3]

\[ [A.2] \Rightarrow \frac{\partial M}{\partial x} (x,t) = V(x,t) = 0 \] [A.4]

As

\( V = \frac{\partial M}{\partial x} \)

\[ [A.3] \Rightarrow \frac{\partial^2 M}{\partial x^2} (x,t) + \rho A(x) \frac{\partial^2 u}{\partial t^2} (x,t) = 0 \] [A.5]

Also \( M(x,t) = EI(x) \frac{\partial^2 u}{\partial x^2}(x,t) \) [2]

(The above equation assumes cross-sections remain plane and is a source of error in the theory as this is not true, only generally for slender members. Three dimensional elements in FEA don’t use this assumption and can be more accurate than classical theory at some instances.)

\[ [A.5] \Rightarrow \frac{\partial^2}{\partial x^2} [EI(x) \frac{\partial^2 u}{\partial x^2}(x,t)] + \rho A(x) \frac{\partial^2 u}{\partial t^2} (x,t) = 0 \] [A.6]

As the beam is uniform \( I(x) \) and \( A(x) \) are constant,

\[ [A.6] \Rightarrow EI \frac{\partial^4 u}{\partial x^4}(x,t) + \rho A \frac{\partial^2 u}{\partial t^2} = 0 \] [A.7]

\[ [A.7] \Rightarrow \frac{EI}{\rho A} \frac{\partial^4 u}{\partial x^4}(x,t) + \frac{\partial^2 u}{\partial t^2} (x,t) = 0 \] [A.8]

where \( \frac{EI}{\rho A} = \sqrt{c} \) or \( c = \frac{EI}{\sqrt{\rho A}} \)

Two initial conditions and four boundary conditions are needed to solve for \( (x,t) \).
As the beam is a free-free suspension then initial conditions are specified as,

\[ u(x,0) = u_0(x) \quad \text{-- IC1} \]

\[ \frac{\partial u}{\partial t}(x,0) = u_0(x) \quad \text{-- IC2} \]

The boundary conditions are therefore;

As no bending moment can be applied to the free produced at the free end then,

\[ M = EI \frac{\partial^2 u}{\partial x^2}(0,t) = 0 \quad \text{-- BC1} \]

\[ M = EI \frac{\partial^2 u}{\partial x^2}(l,t) = 0 \quad \text{-- BC2} \]

Also no shear force can be produced at the free ends then,

\[ V = \frac{\partial}{\partial x}[EI \frac{\partial^2 u}{\partial x^2}(0,t)] = 0 \quad \text{-- BC3} \]

\[ V = \frac{\partial}{\partial x}[EI \frac{\partial^2 u}{\partial x^2}(l,t)] = 0 \quad \text{-- BC4} \]

Using the separation of variables technique to solve,

\[ u(x,t) = U(x)T(t) \quad \text{[A.9]} \]

\[ \text{[A.8]} \Rightarrow c^2 \frac{\partial^4 u}{\partial x^4} T(t) + \frac{\partial^2 u}{\partial t^2} U(x) = 0 \]

\[ c^2 \frac{\partial^4 u}{\partial x^4} \frac{1}{u(x)} = -\frac{\partial^2 u}{\partial t^2} \frac{1}{T(t)} \]

As each side of the equation is only in terms of a single but different variable they must therefore be equal to a constant, as \( x \) cannot vary with \( t \) with a common functional relationship.

Let the variable be \( a \),
\[ c \cdot \frac{\partial^4 u}{\partial x^4} \frac{1}{u(x)} = -\frac{\partial^2 u}{\partial t^2} \frac{1}{T(t)} = a \] \[ \text{[A.10]} \]

\[ \text{[A.10]} \Rightarrow \frac{\partial^2 u}{\partial t^2} + aT(t) = 0 \] \[ \text{[A.11]} \]

\[ \text{[A.10]} \Rightarrow \frac{\partial^2 u}{\partial x^2} - \frac{a}{c^2} u(x) = 0 \] \[ \text{[A.12]} \]

Solving for [A.11]

\[ \frac{\partial^2 u}{\partial t^2} + aT(t) = 0 \]

Let \[ \lambda = \frac{\partial u}{\partial t} \]

\[ \therefore \lambda^2 + a = 0 \]

\[ \therefore \lambda = \pm i\sqrt{a} \]

\[ \therefore T(t) = Ae^{i\sqrt{a}t} + Be^{-i\sqrt{a}t} \] \[ \text{[A.13]} \]

\[ \text{[A.13]} \Rightarrow T(t) = A\cos \sqrt{a}t + B\sin \sqrt{a}t \]

If we let \( a = \omega^2 \),

\[ \therefore T(t) = A\cos \omega t + B\sin \omega t \] \[ \text{[A.14]} \]

from IC1,

\[ T(t) = 0 = A\cos \omega(0) + B\sin \omega(0) \]

\[ \therefore A = 0, \]

\[ T(t) = B\sin \omega t \]

from IC2,

\[ \frac{\partial u}{\partial t}(x, 0) = u_0(x) = B_n \omega \]

Therefore \( B \) or \( \omega \) is 0, for a non-trivial solution \( B = 0 \)

If we assume
\[ U(x) = Ce^{\alpha x} \]

[A.12] \[ \Rightarrow Cs^4e^{\alpha x} - \frac{a}{c^2}Ce^{\alpha x} = 0 \] \[ \text{[A.15]} \]

[A.15] \[ \Rightarrow s^4 - \frac{\omega^2}{c^2} = 0 \]

\[ s = \pm \sqrt[4]{\frac{\omega}{c}}, \pm i\sqrt[4]{\frac{\omega}{c}} \]

\[ :\ U(x) = C_1e^{\sqrt[4]{\frac{\omega}{c}x}} + C_2e^{-\sqrt[4]{\frac{\omega}{c}x}} + C_3e^{i\sqrt[4]{\frac{\omega}{c}x}} + C_4e^{i\sqrt[4]{\frac{\omega}{c}x}} \] \[ \text{[A.17]} \]

\[ :\ U(x) = C_1 \cos \sqrt[4]{\frac{\omega}{c}x} + C_2 \sin \sqrt[4]{\frac{\omega}{c}x} + C_3 \cosh \sqrt[4]{\frac{\omega}{c}x} + C_4 \sinh \sqrt[4]{\frac{\omega}{c}x} \] \[ \text{[A.18]} \]

[A.18] \[ \Rightarrow U(x) = C_1 \cos \beta x + C_2 \sin \beta x + C_3 \cosh \beta x + C_4 \sinh \beta x \] \[ \text{[A.19]} \]

where \[ \sqrt[4]{\frac{\omega}{c}} = \beta \]

\[ \frac{\partial^2 u(x)}{\partial x^2} = -\beta^2 C_1 \cos \beta x - \beta^2 C_2 \sin \beta x + \beta^2 C_3 \cosh x + \beta^2 C_4 \sinh x \] \[ \text{[A.20]} \]

from BC1,

\[ \frac{\partial^2 u(0)}{\partial x^2} EI = -\beta^2 C_1 \cos \beta(0) - \beta^2 C_2 \sin \beta(0) + \beta^2 C_3 \cosh(0) + \beta^2 C_4 \sinh(0) = 0 \]

\[ \therefore \frac{\beta^2 C_2}{EI} = -\frac{\beta^2 C_1}{EI} = 0 \]

\[ C_1 + C_3 = 0 \] \[ \text{[A.21]} \]

\[ \frac{\partial^3 u(x)}{\partial x^3} = \beta^3 C_1 \cos \beta x - \beta^3 C_2 \sin \beta x + \beta^3 C_3 \cosh x + \beta^3 C_4 \sinh x \] \[ \text{[A.22]} \]

from BC3,

\[ \frac{\partial^2 u(0)}{\partial x^2} = -\beta^2 C_1 \cos \beta(0) - \beta^2 C_2 \sin \beta(0) + \beta^2 C_3 \cosh(0) + \beta^2 C_4 \sinh(0) \]

\[ \therefore C_2 + C_4 = 0 \] \[ \text{[A.23]} \]
Combining [A.23],[A.21] and [A.19],

\[ \therefore U(x) = C_1 (\cos \beta x - \cosh \beta x) + C_2 (\sin \beta x - \sinh \beta x) \quad [A.24] \]

from BC2 and [A.24][A.22],

\[ 0 = C_{1n} (\cos \beta L - \cosh \beta L) + C_{2n} (\sin \beta L - \sinh \beta L) \quad [A.25] \]

from BC4 and [A.24][A.20],

\[ 0 = C_{1n} (\cosh \beta L - \cos \beta L) + C_{2n} (\sinh \beta L - \sin \beta L) \quad [A.26] \]

for a non-trivial solution, the determinate of [A.25][A.26] must be zero.

Let,

\[
\begin{align*}
\cos \beta x &= a \\
\cosh \beta x &= b \\
\sin \beta x &= c \\
\sinh \beta x &= d
\end{align*}
\]

\[
\begin{vmatrix}
-a + b & -c + d \\
-c + d & -a + b
\end{vmatrix}
\]

\[ \begin{aligned}
a^2 - 2ab - (-c^2 - d^2) &= 0 \\
-2ab + 1 + 1 &= 0
\end{aligned} \]

\[ \therefore ab = 1 \]

\[ \Rightarrow \cos \beta_n L \cos \beta_n L = 1 \quad [A.27] \]

\begin{align*}
\beta_1 L &= 4.73 \\
\beta_2 L &= 7.85 \\
\beta_3 L &= 10.996 \\
\beta_4 L &= 14.14
\end{align*}

\[ \beta = \sqrt{\frac{\omega}{c}} \]

\[ \therefore \omega_n = (\beta_n L)^2 \left( \frac{EI}{\rho AL^4} \right) \quad [A.28] \]

from [A.24],
\[ C_{2n} = C_{1n} \left( \frac{\cosh \beta_n L - \cos \beta_n L}{\sin \beta_n L - \sinh \beta_n L} \right) \]  

[A.29]

The total solution for the shape of the beam at displacement \( x \) and time \( t \) for a particular frequency is;

\[ U(x, t) = U(x)T(t) \]

\[ U(x, t) = U(x)[B_n \sin \omega_n t] \]

when \( B_n = \frac{u_0(0)}{\omega_n} \)

The position of the beam at any time is a sum of all the modes to infinity. As well \( u_0(0) \) needs to be known.

\[ u(x, t) = C_{1n} \left[ \sin \beta_n x + \sinh \beta_n x + a_n (\cos \beta_n x + \cosh \beta_n x) \right] \]

where \( a_n = \frac{\sin \beta_n L - \sinh \beta_n L}{\cosh \beta_n L - \cos \beta_n L} \)  

[A.30]
Appendix B.  Matlab Scripts.

% Load mode shape data from file
load nocrackmode

% Transform mode shape data to appropriate values
gg(2:64,:) = ndata;
dd = ndata(63,:);
gg(1,:) = dd;
ndata = gg(1:63,:);

% Create a row vector of all ones
pm1 = ones(63,1);

% Assign a postive value to pm1 as phase is 180 degrees, assign negative
% value if phase is 0 degrees.
for i =1:63;
    if ndata(i,2)<20;
        pm1(i,1) = -1;
    end;
end;

% Account for phase differences by multiplying adjusted row vector by
% original data
nop1n = pm1.*ndata(:,1);
figure(11);

% plot phase adjusted mode shapes
plot(nop1n);

% Average the values across the ensemble of measurements over beam width,
% to help reduce noise.

b = 1:21;

a = nop1n(1:21,1);

a(:,2) = flipud(nop1n(22:42,1));

a(:,3) = nop1n(43:63,1);

a = a';

aa = mean(a);

aa1n = aa';

% Normalise mode shape

aa1n= aa1n./max(aa1n);

plot(aa1n)

for i = 1:63

    ph1(i,1) = cos((ndata(i,2))*pi/180);

end;

phas = ph1.*ndata(:,1);

figure(12);

plot(phas);

% Create vectors for x-axis

x = 1:0.1:21;

x1 = 1:0.3:21;

e = 1:0.1:20.9;

f = 1:0.1:20.8;

g = 1:0.1:20.7;

% Apply a hermite piece wise cubic polynomial to the mode shape data

yy1n = pchip(b,aa1n);
% Sample the piece wise hermite polynomial at 3 times original samples
yy1n = ppval(yy1n,x1)

% Apply a cubic spline function to the new samples, with enforced zero
% curvature boundary conditions at ends
yy1n1 = csape(x1,yy1n,'v');

% Sample the cubic spline at 10 times original sampling.
yy1n = ppval(yy1n1,x)

% calculate analytical first derivative of cubic spline fitted function to
% find slope
slope1 = fnder(yy1n1);
slope1n = ppval(slope1,x);

% calculate analytical second derivative of cubic spline fitted function
% to find curvature
cature1 = fnder(yy1n1,2);
cature1n = ppval(cature1,x);

% calculate analytical third derivative of cubic spline fitted function to
% find change in curvature
cchcture1 = fnder(yy1n1,3);
cchcture1n = ppval(chcture1,x);

figure(1);
% plot all functions on same graph
plot(x,yy1n,e,slope1n(1,1:200),f,cature1n(1,1:199),g,chcture1n(1,1:198));
title('mode shape and all derivatives without crack');
xlabel('element');
ylabel('magnitude');
figure(2);
% plot averaged mode shapes
plot(x,yy1n);
title('First bending mode without crack');
xlabel('element');
ylabel('relative displacement');
figure(3);
% plot slope of cubic fitted mode shape
plot(e,slope1n(1,1:200));
title('First bending mode slope without crack');
xlabel('element');
ylabel('gradient');
figure(4);
% plot curvature of cubic fitted mode shape
plot(f,cture1n(1,1:199));
title('First bending mode curvature without crack');
xlabel('element');
ylabel('curvature');
figure(5);
% plot change in curvature of cubic fitted mode shape
plot(g,chcture1n(1,1:198));
title('First bending mode change in curvature without crack');
xlabel('element');
ylabel('change in curvature');
Appendix C.  **Hammer test.**

The global polynomial curve fitting of the mode shapes from the shaker excitation of the structure produced mode shapes which displayed not physically viable behaviour and that only with the use of quadrature picking methods were adequate mode shapes able to be constructed. It was hypothesised that the localised mass from the accelerometer which was moved around the structure during the modal analysis and changes the structure each time. This change caused by the small localising of the mass from the accelerometer and the very light damping of the structure caused the moving of the natural frequencies for each measurement slightly. This moving of the natural frequencies caused a large change in the measured FRF’s at the globally set peak value, and with the step flacks of the FRF’s because of the very light damping would cause bad mode shape construction. To elevate this problem hammer excitation could have been used or artificial introducing of damping to the structure. The hammer excitation was already ruled out as the method of excitation due to wanting to introduce larger amounts of energy into the system that only shaker excitation could produce. The introduction was also not deemed available to be produced at this time of experimentation. So quadrature picking methods were used with the curse offset to the next capture data point from the peak in the natural frequencies as the slight movement of the peak value would have a much lesser effect on the values slightly away from the peak, than at the position of the peak itself.

Hammer test excitation was conducted to validate that the hypothesis of the localisation of the accelerometer mass and the very light damping of the system, was the cause of the poor curve fitting was undertaken. The fitting curve using global curve fitting and the constructed mode shape is shown in Figure C1, and as can be
seen they are of very good quality. Table C1 shows the natural frequencies and
damping produced for the global polynomial fitting and can be seen to be the same as
was found with the shaker excitation and quadrature picking. This therefore validates
the quadrature curve fitting method used in the analysis of the thesis and also
validates the hypothesis that the localising of the mass of the accelerometer in
conjunction with the very light damping of the structure did not allow adequate global
polynomial curve fitting of the mode shape measured with shaker excitation.

![Figure C1 (First four bending modes with hammer testing)]
Table C1 (Constructed modes shapes from hammer testing)

<table>
<thead>
<tr>
<th>Mode Shape</th>
<th>Curve Fitting Type</th>
<th>EMA(Hz)</th>
<th>Damping(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GPO</td>
<td>104</td>
<td>1.59</td>
</tr>
<tr>
<td>2</td>
<td>GPO</td>
<td>290</td>
<td>0.535</td>
</tr>
<tr>
<td>3</td>
<td>GPO</td>
<td>562</td>
<td>0.306</td>
</tr>
<tr>
<td>4</td>
<td>GPO</td>
<td>931</td>
<td>0.174</td>
</tr>
<tr>
<td>5</td>
<td>GPO</td>
<td>1.38E3</td>
<td>0.171</td>
</tr>
</tbody>
</table>
Appendix D.  Critical Review of Software Systems

The invention of the Fast Fourier Transform, FFT, brought about a fast, efficient and practical way to measure the vibrational properties of systems. With this has come a great deal of post measurement analysis tools along with many proprietary measurement systems. All the processing software in some form or another is based upon the transformation of experimental measurements into an idealised mathematical model to either characterise the system or for future predictions of its behaviour under certain conditions.

The critical review of the three main software packages that were utilised within the thesis are presented here. Data Physics’ ACE Signal Calc. is a data capturing device and is where the initial measurements are captured and initial processing is undertaken. Further processing of the systems vibrational parameters is then done using the Vibrant Technologies ME-scope software. The main function of this software is the, very common and useful technique, of defining a system into a set of mode shapes which can be used to define its deflection at a certain frequency. Finally the review of FEM-Tools is undertaken, with this program offering the ability to test the correlation between the measured vibrational properties of a system and those which are derived from an analytical finite element model of the target system. This program also offers the ability to update the parameters of the finite elements such as to match the responses for the analytical finite element model and the measured systems responses.

For a purely objective critical review of the software used, one should have a working knowledge of other peer software packages available. The author however only has a good working knowledge of the software packages discussed below of
each of the packages for their specific analysis utilisation. This should be noted in the conclusions that any reader derives from the ideas presented.

D.1 ACE Signal Calc.

ACE Signal Calc. is a two channel data acquisition system which is provided by Data Physics. The graphical interface of ACE Signal Calc. provides a user friendly platform for which the data capture is easily navigated through by the occasional user. The ACE system has a few features which should be noted by the user before use of the system which is not readily discussed in the proprietary literature of the system.

A wide range of signals are available for processing and exporting for further analysis, however there is no ability to simultaneously send the $H_1$ and $H_2$ FRF estimate. It is only able to be defined in the initial measurement setup by the way each measurement signal is numbered relative to the other in the two channel input system. The inclusion of the ability to have the choice of either the $H_1$ or $H_2$ estimate during the post processing of the data would be a desirable update, and any user should be aware of the inability of deciding which estimate to use after the measurements have been done to minimise the hardship of manually manipulating the date to obtain either.

Another fundamental short fall that the ACE system has, is there existing no graphical representation of the structure and animation of the structure, as is available in other data acquisition systems such as the Bruel and Kjaer PULSE system. The inclusion of this feature would also be a welcome addition to the software package.

The underlying offered abilities of the ACE system are quite comprehensive and would be a very adequate data acquisition system for everyday use if the program lived up to the claims of its abilities. Unfortunately the system often does not live up to its claimed abilities in use, this may be isolated to the use of this one actual system.
which was used singularly however. Results obtained for the hammer excitation mode of measurement setup was particularly erratic in nature and unable to produce stable, reliable, repeatable results. The trigger of the system for the hammer input seemed to be to blame for the poor ability during the data capture in this mode, however the results obtained during the shaker excitation measurement capture also produced erratic results at times. During the hammer excitation data capture, there also seemed to exist a memory caching problem with the system becoming so slow during the measurement process that it is unusable.

If diligent user observance of the results obtained from the acquisition system is undertaken then adequate results are able to be obtained while using the ACE Signal Calc. but until entire confidence can be had in the results obtained at all times, there will be a sense of the systems inability to be a high class data capture program.

D.2 ME-scope

ME-scope is probably the most highly developed data processing system of the three systems reviewed here, as it is a very stable and user friendly interface and seems to offer a wide range of functions for the user. The curve fitting of the input data is still not a process of point and click, as was highlighted in the poor curve fitting of the initial results obtained during the curve fitting of the experimental data which was discussed earlier in chapter 3. With a good physical understanding of the processing involved though, appropriate results are able to be obtained by the user. This program could however benefit from a better interfacing method of processing the curve fit data into a high powered scientific programming tool such as matlab. The inability of a simple interfacing method for both the export and then re-import of data to an interface such as this is a draw back on an otherwise good system.
D.3 FEM-Tools

FEM-Tools brings together the results obtained from experimental measurements and those from an analytical finite element model. Therefore it is necessary to have a good working knowledge of the physical mechanics of the structure as well as the ability to drive the analysis software adequately. Unfortunately as is often common with FEA analysis, a high quality grasp of only one of these principles and often the results helping to conspire the user to believe there ability to bring the two sets of knowledge together are better than what they are in reality. This is highlighted in the infamous fact that it is estimated that 90% of finite element models worldwide give unacceptable inaccurate results [4]. FEM-Tools was able to provide a robust updating program with only the minimal of programming bugs which effected its use. It is limited much more by the users ability than the functionality of the system, which provides probably the greatest user friendly system on offer here.
Appendix E. Supplementary CD Data Tree

Figure E.1 (Supplementary CD Data Tree)