Salter's Method

Aim of this note is to explain with a simple example of a tuned absorber, the use of the Mobility Method, also know as Salter's Method, to identify the Mobility in a 2-DOF system. Mobility M is defined as the ratio between velocity and force, $M = \frac{\dot{x}}{F}$

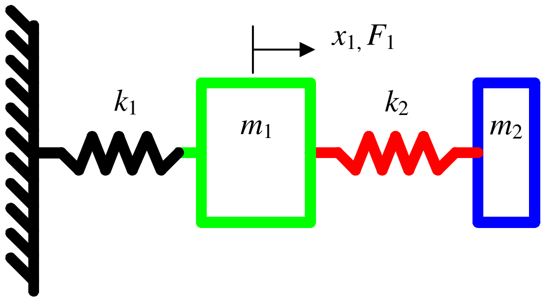
For a mass we have: $F = m\ddot{x} = m(j\omega\dot{x})$ $\quad M_m = \frac{\dot{x}}{F} = \frac{1}{j\omega m} = \frac{-j}{\omega m}$

For a spring we have $F = kx = k\frac{\dot{x}}{j\omega}$ $\quad M_k = \frac{\dot{x}}{F} = \frac{j\omega}{k}$

Impedance Z is defined as $Z = \frac{F}{\dot{x}} = \frac{1}{M}$, then $Z_m = j\omega m$, $Z_k = \frac{-jk}{\omega}$

EXAMPLE (Tuned absorber)

We want to get the Mobility $M = \dot{x}_1 / F_1$ for the following system:

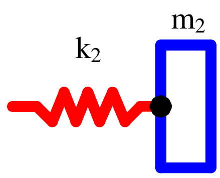


Data: $k_2 = k_1 = 100 \ N/m$, $\quad m_1 = 10 \ Kg$, $\quad m_2 = 4 \ Kg$

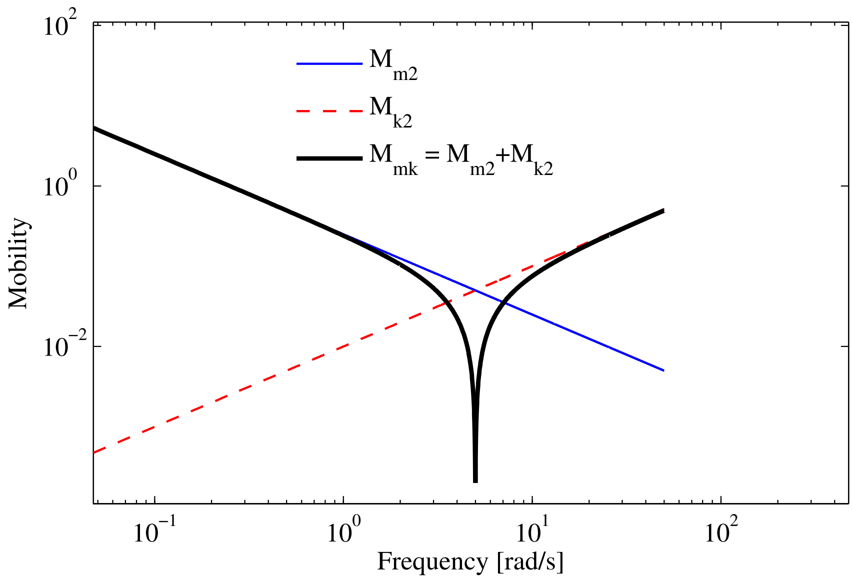
In all the following Figures the absolute values of mobility and impedance are plotted.

STEP 1

At first we joint the mass (blue) and the spring (red), they share the same force, then the total Mobility is the sum of the single mobilities.



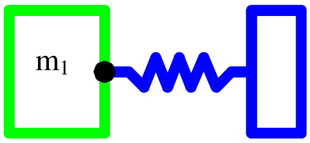
$$M_{m_2} = \frac{1}{j\omega m_2} = \frac{-j}{\omega m_2} \qquad M_{k_2} = \frac{j\omega}{k_2} \qquad M_{mk} = M_{m_2} + M_{k_2}$$

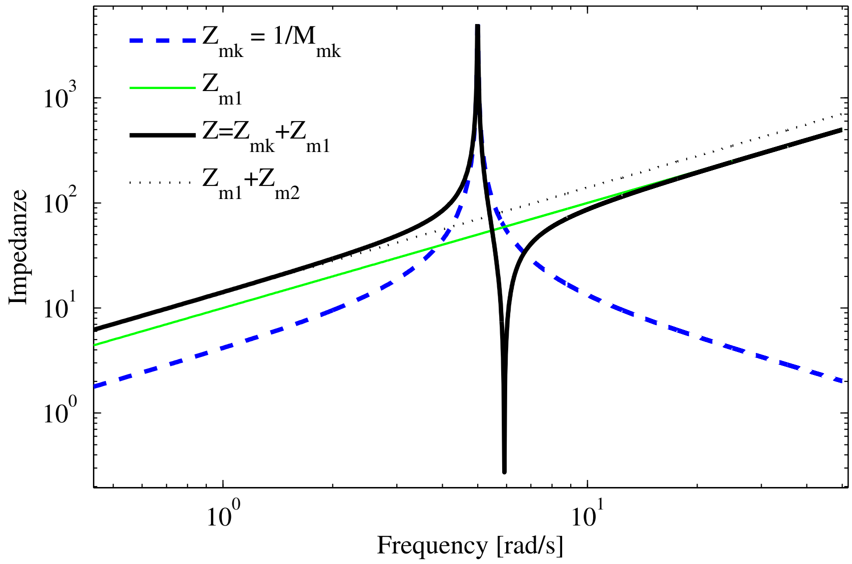


Note that the minimum is at the crossing of the mass and spring mobilities since they are 180º out-of-phase, this happen when $\frac{-j}{\omega m_2} = \frac{j\omega}{k_2} \rightarrow \omega = \sqrt{\frac{k_2}{m_2}}$ (with the data we get $\omega = 5$ rad/s). If the spring were fixed to a base, the total impedance is the sum of the single impedances, since they would share the same motion. The mobility would look like the inverse of the mobility plotted before, giving the well-know impedance for a spring-mass system with the maximum at the resonance $\omega = (k_2 / m_2)^{1/2}$

STEP 2

Now, the system analysed above (mass-spring in blue) has to be joined to the second mass (in green), this time they share the same motion, at the connection point. Then the total Impedance is the sum of the single impedances



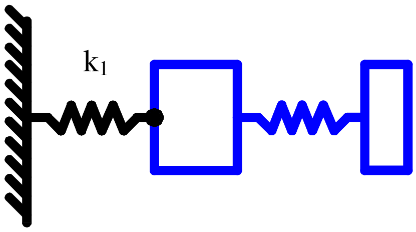


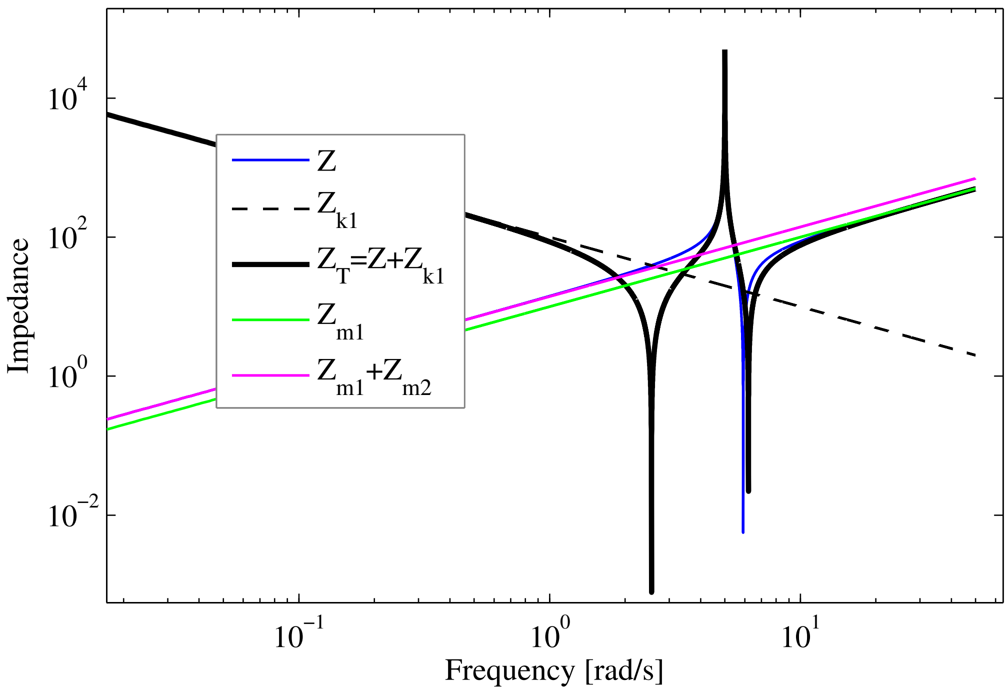
The anti-resonance happens at the frequency where the $Z_{mk} = Z_{m_1}$,

$\left(\frac{j\omega}{k_2}-\frac{j}{\omega m_2}\right)^{-1} = j\omega m_1$ to get $\omega=\sqrt{\frac{k_2}{m_1}+\frac{k_2}{m_2}}$ (ω=5.9161 rad/s)

STEP 3

Finally, the new system (mass-spring-mass in blue) has to be joined to the base spring (in black), they share the same motion, at the connection point. Then the total impedance Z_T is the sum of the single impedances

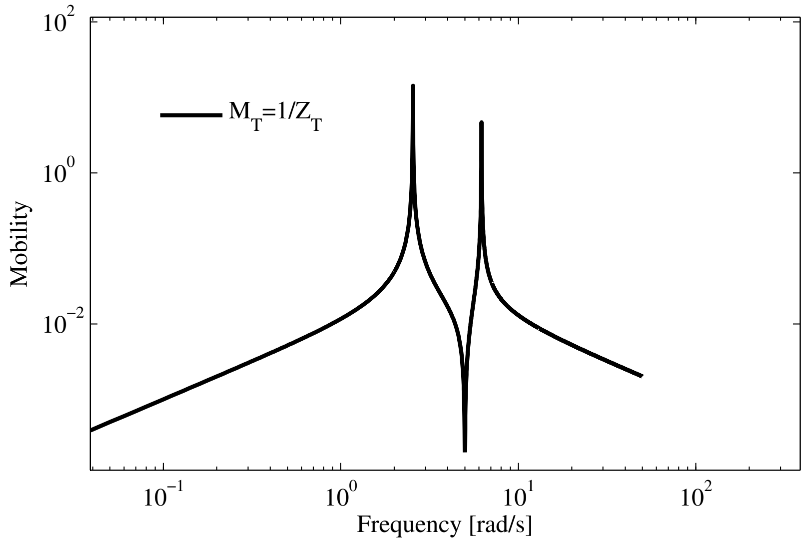




The first minimum in the impedance happens when $Z_{m_1} + Z_{m_2} \simeq Z_{k_1}$, $j\omega(m_1 + m_2) \simeq \dfrac{k_1}{j\omega}$

to get $\omega \simeq [k_1 / (m_1 + m_2)]^{1/2}$ (ω=2.67 rad/s)

To get the Mobility $M_T = \frac{\dot{x}_1}{F_1}$, just invert the Impedance Z_T



Electric equivalence

There problem can be seen using the electric equivalence between electric and mechanical systems. The reason of this equivalence is that the equations are formally the same:

MECHANICAL: $m\ddot{x} + c\dot{x} + kx = F$ rewritten as $m\frac{dv}{dt} + cv + k\int vdt = F$

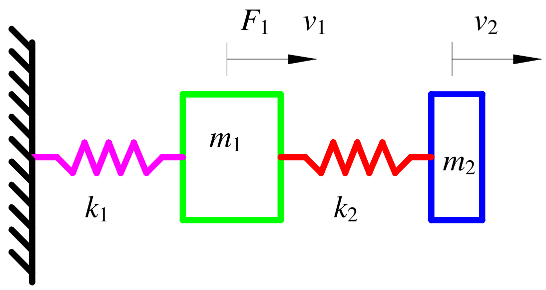
ELECTRIC: $L\frac{di}{dt} + Ri + C^{-1}\int idt = V$

Then we have the following equivalence:

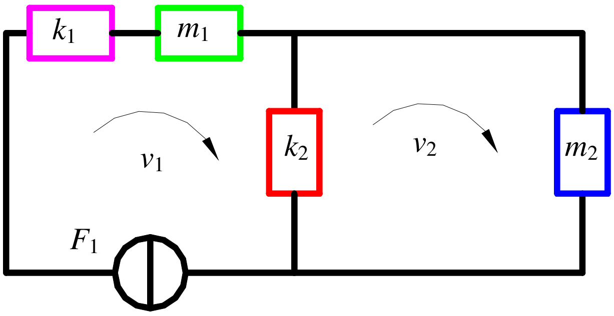
Force (F) = Voltage (V), Velocity (v) = Current (i), Mass (m) = Inductance (L), Damper (c) = Resistance (R), Spring (k) = Capacitance (C)

Impedances (mechanical *F/v)* and electrical (*V/i)* are then formally identical. It can be seen that in the same way the energy, mechanical or electrical, is dissipated in the damper or in the resistance and it is stored in the spring or in the capacitance. For the system in the example the equivalent circuit is the following:

Mechanical system:



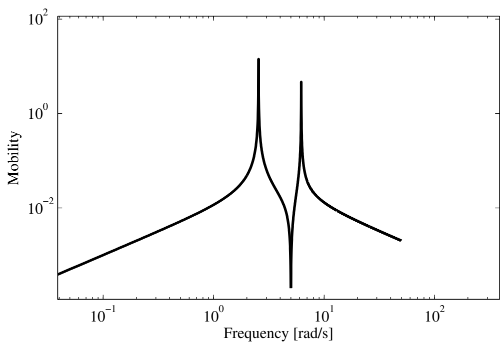
Equivalent electrical system:



Applying the mesh current method, we can write the equation for the 2 meshes (2-DOF) :

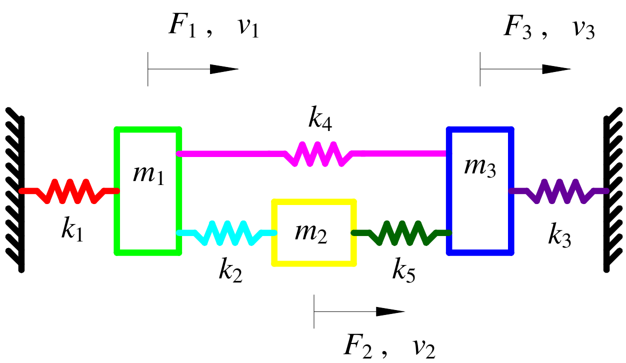
$$\begin{cases} F_1 - (Z_{k1} + Z_{m1})v_1 - Z_{k2}(v_1 - v_2) = 0 \\ Z_{k2}(v_2 - v_1) + v_2 Z_{m2} = 0 \end{cases} \quad \text{to get:} \quad M = \frac{v_1}{F_1} = \left(Z_{k1} + Z_{m1} + Z_{k2} - \frac{Z_{k2}^{\;2}}{Z_{k2} + Z_{m2}} \right)^{-1}$$

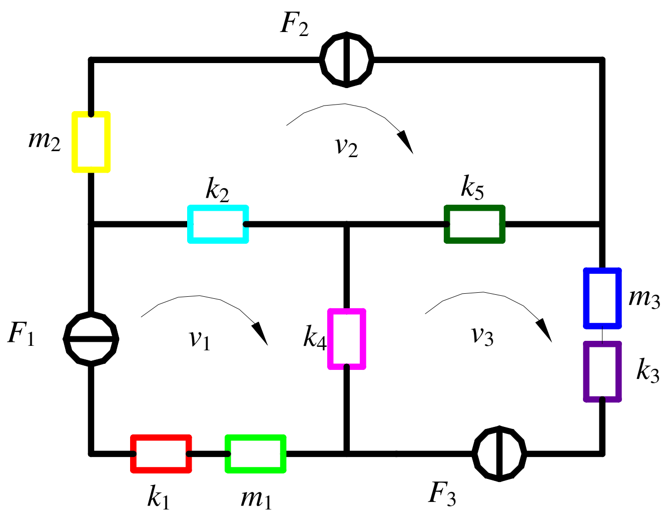
The same result as before



EXAMPLE

Following the same procedure we can write the equivalent circuit for the mechanical system below:





It is easy to see that the energy stored in the spring (capacitance) purple is proportional to $(v_1 - v_3)$, in the spring (capacitance) cyan is proportional to $(v_1 - v_2)$, in the spring (capacitance) dark green is proportional to $(v_3 - v_2)$. In case dampers that react to the ground are introduced in the mechanical system, the equivalent electric resistances will be added in the circuit, in series with inductances (i.e. masses in the mechanical system). This is because the energy dissipated is proportional to the absolute motion and not to the relative motion as in the case of the energy stored in the springs.