Epicyclic Gears

Aim of this note is to explain the direct method to solve problems with Epicyclic Gears The Epicyclic Gear first analysed here have the following components:

- 2 main shaft, input and output with angular velocity ω_i and ω_o respectively.
- A planet with 2 gears, G_{p1} and G_{p2}
- 1 arm (R) connected to the input or output shaft
- 2 main gears G_1 and G_2 that are in contact with G_{p1} and G_{p2} respectively.

Procedure

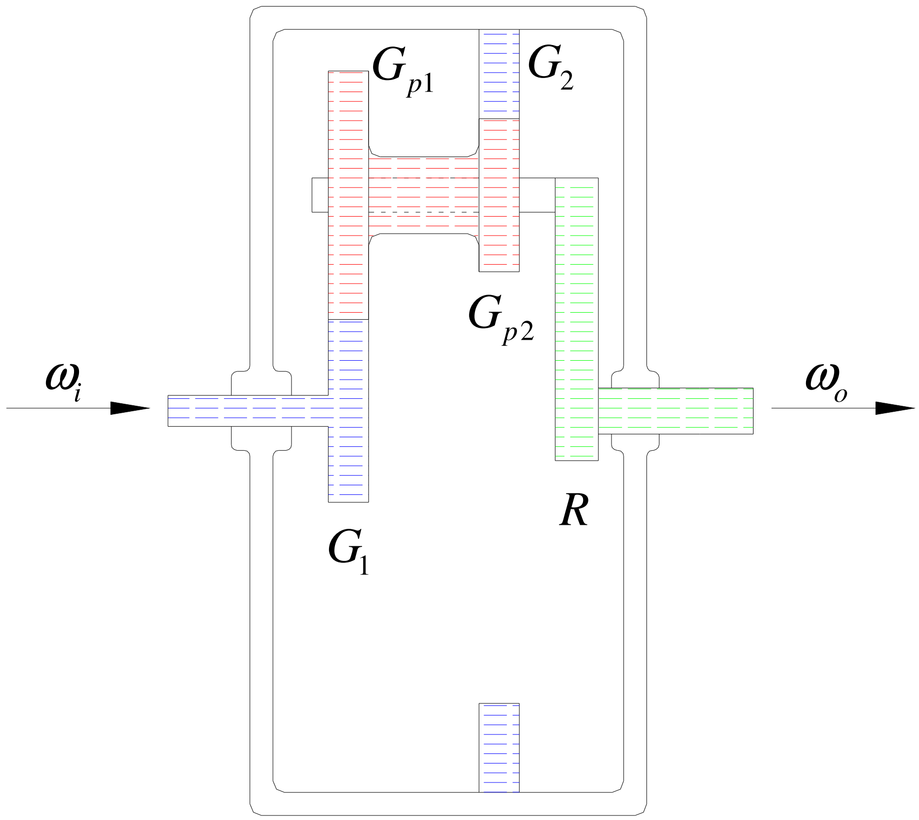
1) Write down the two gear trains that share the arm (R) starting from the input shaft, usually the planet gear is the middle one. In case there is a third gear there will be 3 trains and then 3 equations. For example we can have [G_1-G_{p1}-R]. To define a train, consider that it has always the arm R , a gear of the planet and a main gear. Just eliminate one main gear per time.
2) The problem has 2 unknowns, the ratio ω_i / ω_o and ω_p, we need to write 2 equations:

   - The first equation reads: $\omega_p = \omega_R \left(1 \pm \frac{N_1}{N_{p1}}\right) \mp \omega_1 \frac{N_1}{N_{p1}}$

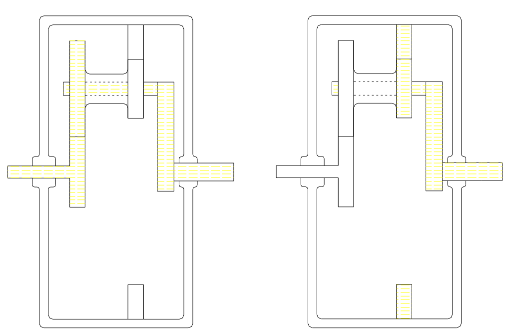
   - The second equation reads: $\omega_p = \omega_R \left(1 \pm \frac{N_2}{N_{p2}}\right) \mp \omega_2 \frac{N_2}{N_{p2}}$

ω_p is the angular velocity of the planet, It will be the sum of different components, this is because the planet is rotating around a fixed axis but also around his own axis, for this reason it is called "planet" gear.

ω_R is the angular speed of the arm, equal to the input angular speed ω_i or output angular speed ω_o, depending on its location. ω_1 and ω_2 are the angular speed of the two gears G_1 and G_2. N is the gears' teeth number, the sign $\pm$ depends on the type of gear and , '- + in sequence' is for external gears. Before explaining the meaning of the equations, let's show an EXAMPLE 1. We have the following epicyclic gear:



The two trains are:



$[G_1\text{-}G_{p1}\text{-}R]$ $[G_2\text{-}G_{p2}\text{-}R]$

In this case the following apply:

$\omega_1 = \omega_i$

$\omega_R = \omega_o$

$\omega_2 = 0$

Then the system of two equations to solve by elimination of ω_p simply reads:

$$\omega_p = \omega_o \left(1 + \frac{N_1}{N_{p1}}\right) - \omega_i \frac{N_1}{N_{p1}}$$

$$\omega_p = \omega_R \left(1 - \frac{N_2}{N_{p2}}\right)$$

Quick explanation

Let's refer to the first equation. The motion of the planet is a revolution around a fixed axis plus a rotation around its own axis. The planet is driven by the arm *R* that is rotating at ω_R, so the arm gives the planet's revolution angular velocity $\omega_{rev} = \omega_R$. The rotation ω_{rot} around its axis is given by the connection with the gear G_1 that is also rotating at its own rotation speed ω_1, the direction depends on the location of the gear. Then the total angular velocity of the planet can be written as the sum of two components:

$$\omega_p = \omega_{rev} + \omega_{rot}$$

Revolution motion: $\omega_{rev} = \omega_R$

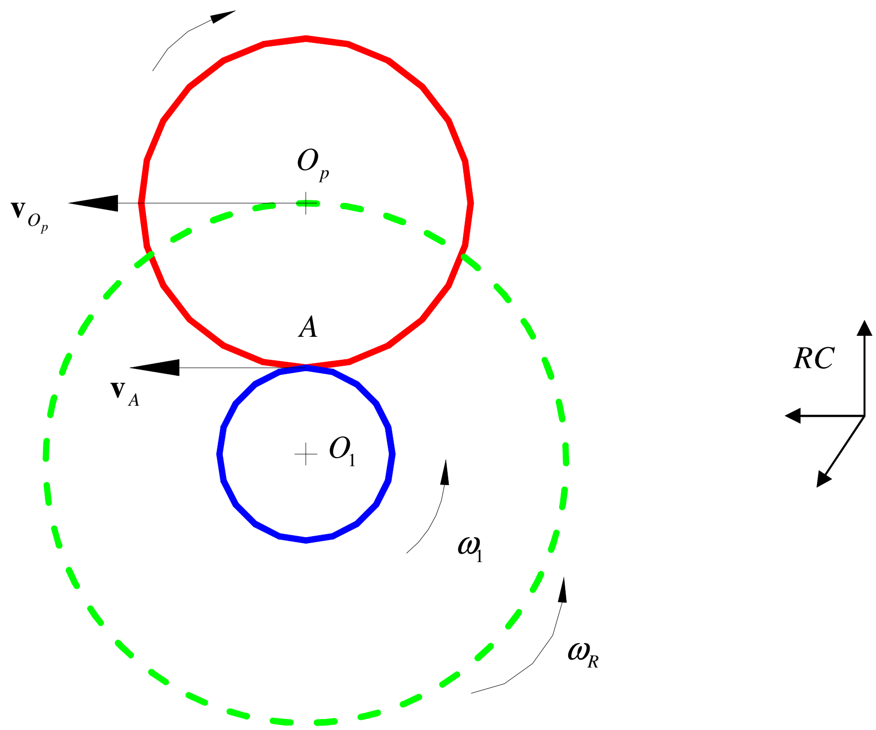
Rotation on his own axis: $\omega_{rot} = \left(\pm\omega_R \mp \omega_1\right)\frac{R_1}{R_{p1}}$

Arranging and considering that $\frac{R_1}{R_{p1}} = \frac{N_1}{N_{p1}}$ we get the formula:

$$\omega_p = \omega_R \left(1 \pm \frac{N_1}{N_{p1}}\right) \mp \omega_1 \frac{N_1}{N_{p1}}$$

Rigorous explanation

Considering the velocities of the contact point A and of the centre of the planet O_p according to the sign convention RC



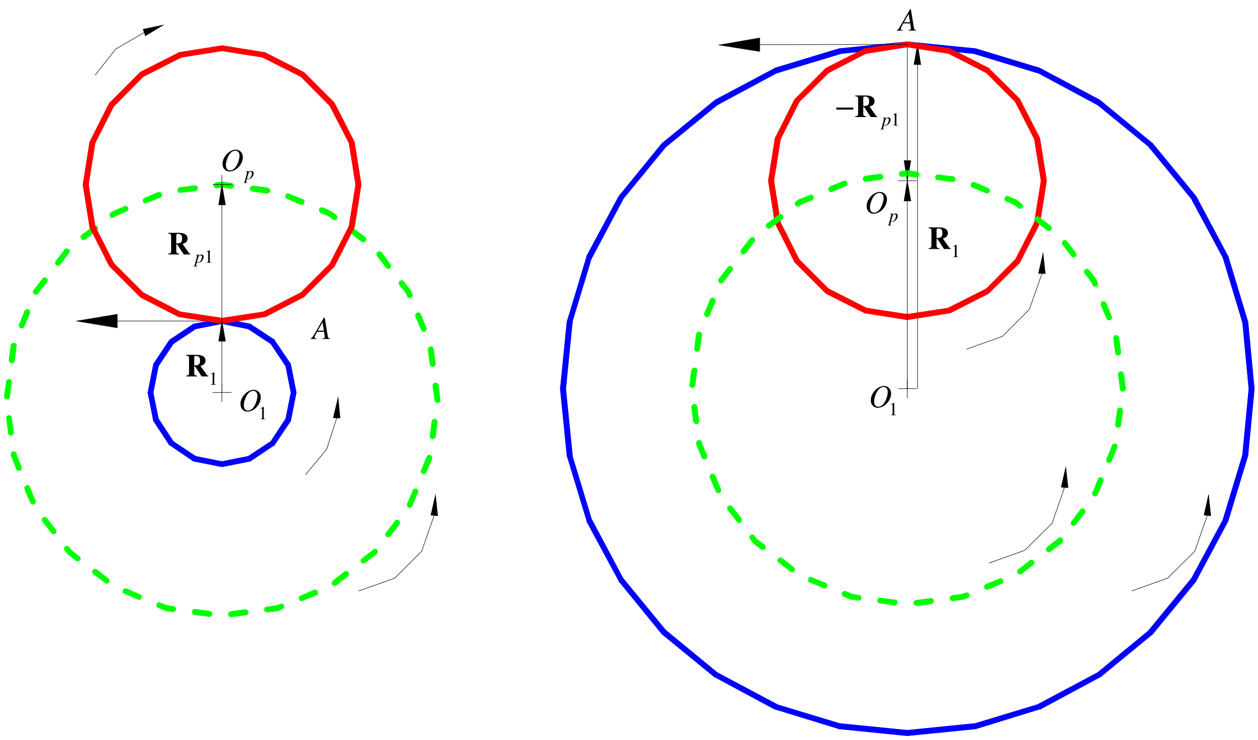
The velocity of point A where G_1 is in contact with Gp_1, can be written considering it as belonging to the gear G_1 ($\mathbf{v}_A|_1$) or to the planet ($\mathbf{v}_A|_p$) (the velocities are positive in left direction). Then we have:

$$\mathbf{v}_A|_1 = \boldsymbol{\omega}_1 \times \overrightarrow{O_1 A}$$

$$\mathbf{v}_A|_p = \mathbf{v}_{O_p} + \boldsymbol{\omega}_p \times \overrightarrow{O_p A}$$

with $\mathbf{v}_{O_p} = \boldsymbol{\omega}_R \times \overrightarrow{O_1 O_p}$

If the main gear G_1 has internal or external teeth, we get different solutions:



External teeth: $\overrightarrow{O_1O_p} = \mathbf{R}_{p1} + \mathbf{R}_1$ Internal teeth: $\overrightarrow{O_1O_p} = \mathbf{R}_1 - \mathbf{R}_{p1}$

$\overrightarrow{O_pA} = -\mathbf{R}_{p1}$ $\overrightarrow{O_pA} = \mathbf{R}_{p1}$

Then, writing the moduli of the vectors (with the sign convention of *RC*):

External teeth gear:

$$v_{O_p} = \omega_R(R_{p1} + R_1)$$

$$v_A\big|_1 = \omega_1 R_1 \quad , \quad v_A\big|_p = \omega_R(R_{p1} + R_1) - \omega_p R_{p1}$$

Internal teeth gear:

$$v_{O_p} = \omega_R (R_1 - R_{p1})$$

$$v_A\big|_1 = \omega_1 R_1 \qquad , \qquad v_A\big|_p = \omega_R (R_1 - R_{p1}) + \omega_p R_{p1}$$

Since $v_A\big|_1 = v_A\big|_p$,

we get $\omega_p = \omega_R \left(1 \pm \frac{R_1}{R_{p1}}\right) \mp \omega_1 \frac{R_1}{R_{p1}}$

usually written in terms of teeth number instead of the radii:

$$\omega_p = \omega_R \left(1 \pm \frac{N_1}{N_{p1}}\right) \mp \omega_1 \frac{N_1}{N_{p1}}$$

(+)(-) for external teeth, (-)(+) for internal teeth

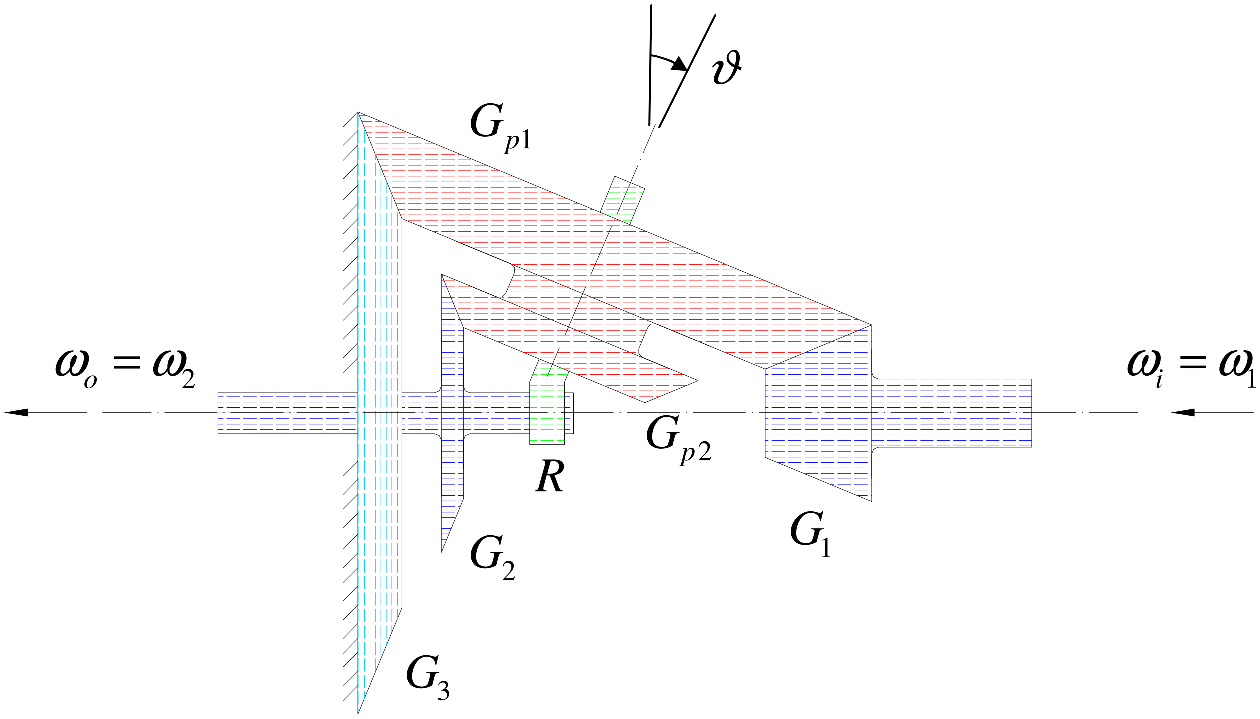
We can notice that, considering a reference position of the planet on the highest location:

- External teeth gear: the contact point *A* is on the bottom of the planetary gear
- Internal teeth gear: the contact point *A* is on the top of the planetary gear

It follows that in case we have an external teeth gear but it is conical and then the contact point *A* is on the top of the planetary gear, the formula to use is the same as in case with internal teeth. (see EXAMPLE 2 and 3)

EXAMPLE 2

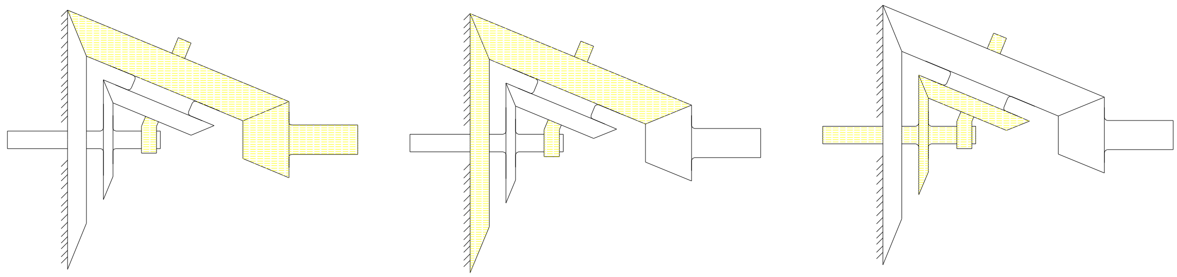
Consider now a planetary gear with 3 main gears shown in the figure below:



The Epicyclic Gear has then following components:

- 2 main shaft, input and output with angular velocity ω_i and ω_o respectively.
- A planet (in red) with 2 gears, G_{p1} and G_{p2}
- 1 arm (R) (in green)
- 3 main gears G_1 , G_2 and G_3 (in blue colour) that are in contact with the gears of the planet. This time G_{p1} is in contact with 2 main gears, G_1 and G_3

As before we have to define the trains, this time we have 3 trains since there are 3 main gears:



1) $[G_1 - G_{p1} - R]$ 2) $[R - G_{p1} - G_3]$ 3) $[R - G_{p2} - G_2]$

The problem has 2 unknowns as before, the ratio ω_i / ω_o and ω_p, plus a third unknown that is ω_R, this is because here the arm R is free to rotate while before was directly connected to the input shaft. Now we need to write 3 equations corresponding to the 3 trains above:

1) The first equation reads: $$\omega_p = \omega_R\left(1+\frac{N_1}{N_{p1}}\right) - \omega_1 \frac{N_1}{N_{p1}}$$

2) The second equation reads: $$\omega_p = \omega_R\left(1-\frac{N_3}{N_{p1}}\right) + \cancel{\omega_3 \frac{N_3}{N_{p1}}}$$

3) The second equation reads: $$\omega_p = \omega_R\left(1-\frac{N_2}{N_{p2}}\right) + \omega_2 \frac{N_2}{N_{p2}}$$

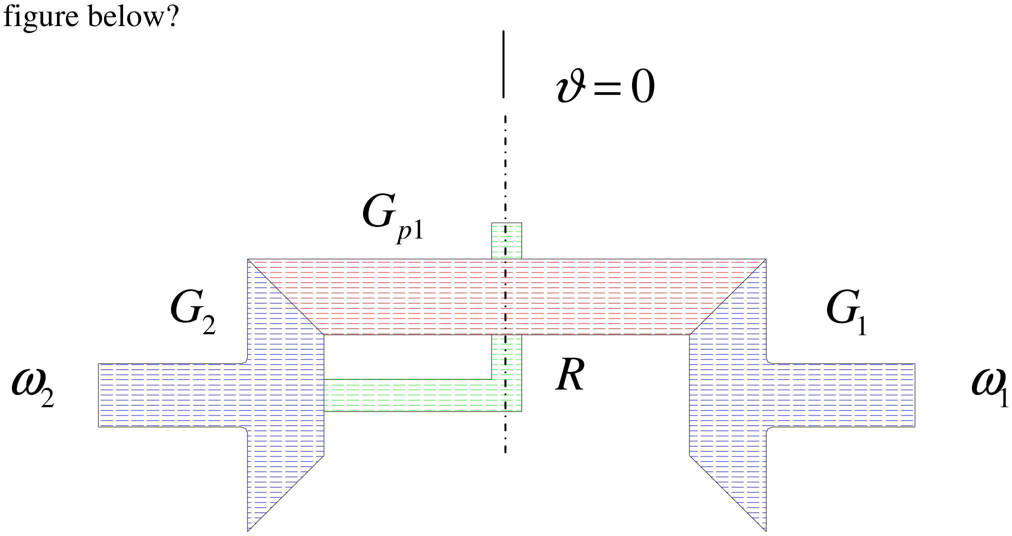
Note that $\omega_3 = 0$ since the gear G_3 is fixed.

The signs in the equations 2 and 3 are because the gears G_2 and G_3 behave like external gear (i.e. the contact is on the top of the planetary gears).

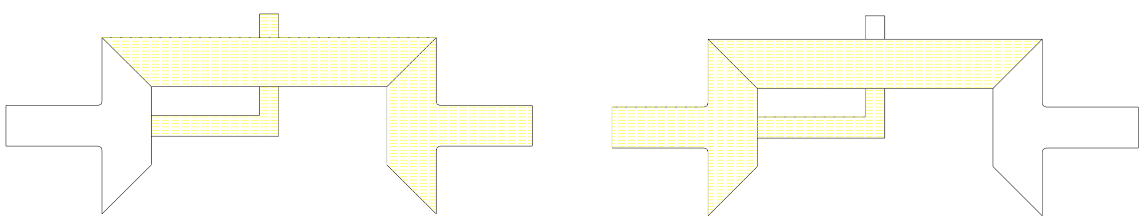
We can get the ratio ω_1 / ω_2 solving the system of the three equations.

EXAMPLE 3

How should we deal with a case in which the planet axis is vertical in his highest location as in figure below?



We can see this case as a limit for the angle $\vartheta \to 0$, this angle is defined as in the figure before, then G_1 will behave as an external teeth gear and G_2 as an internal teeth gear as in the limit the contact point is on the top of the planetary gear.



1) $[G_1 - G_{p1} - R]$ 2) $[G_{p1} - R - G_2]$

1) The first equation reads: $$\omega_p = \omega_R \left(1 + \frac{N_1}{N_{p1}}\right) - \omega_1 \frac{N_1}{N_{p1}}$$

2) The second equation reads: $$\omega_p = \omega_R \left(1 - \frac{N_2}{N_{p1}}\right) + \omega_2 \frac{N_2}{N_{p1}}$$

A common application of this planetary gear is the differential gear in a car, in this case, $N_1 = N_2$ and there are some interesting situations:

If $\omega_1 = \omega_2 \rightarrow \omega_R = \omega_p = \omega_1$, the planet just spin around but no rotation around his own axis

If $\omega_2 = 0 \rightarrow \omega_R = \dfrac{\omega_1}{2}$