The meaning of the Convolution

This note aims to explain the meaning of the Convolution between two functions.
The convolution of two functions $x(t)$ and $h(t)$ is defined as:

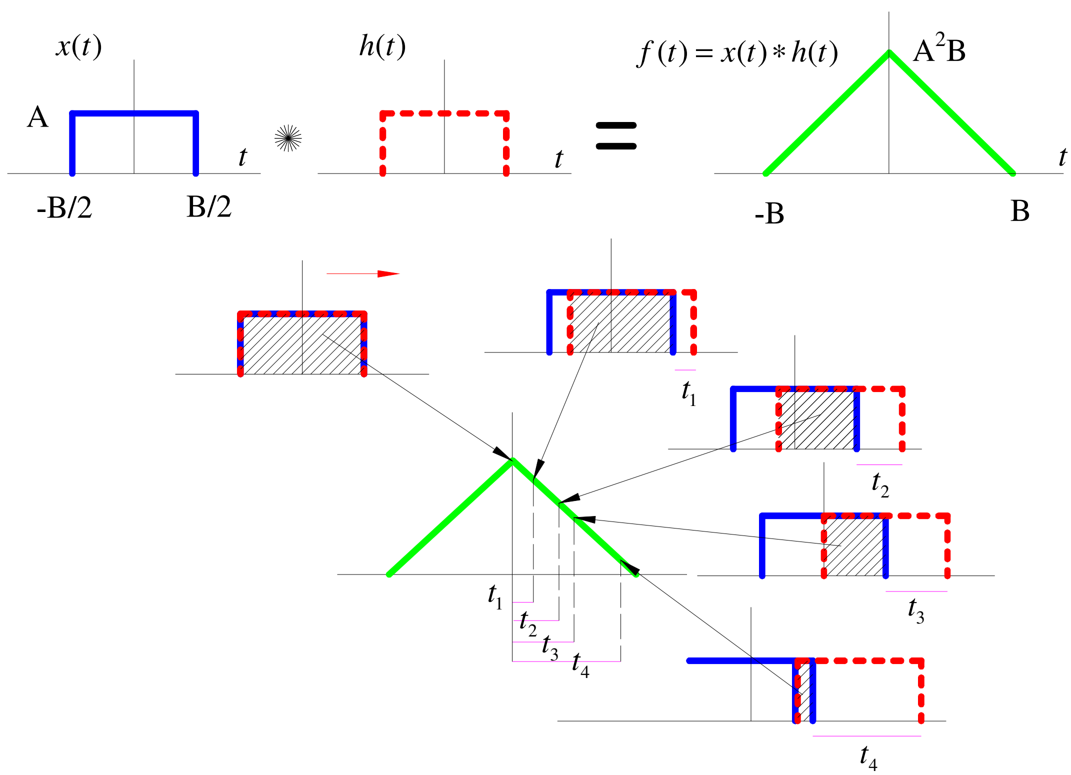
$$f(t) = \int_{-\infty}^{\infty} x(\eta) \cdot h(t-\eta) d\eta$$

To understand the meaning of the convolution we can break it down into the following steps:

1) x and h are given as function of a dummy variable η
2) Transpose of the functions: $h(\eta) \rightarrow h(-\eta)$
3) Add an offset t which allows $h(t-\eta)$ to slide along the η axis in the right direction.
4) The value at a fixed t_x is given by the area of the curve resulted by the product of the two functions $x(\eta) \cdot h(t_x - \eta)$, i.e. $f(t_x) = \int_{-\infty}^{\infty} x(\eta) \cdot h(t_x - \eta) d\eta$

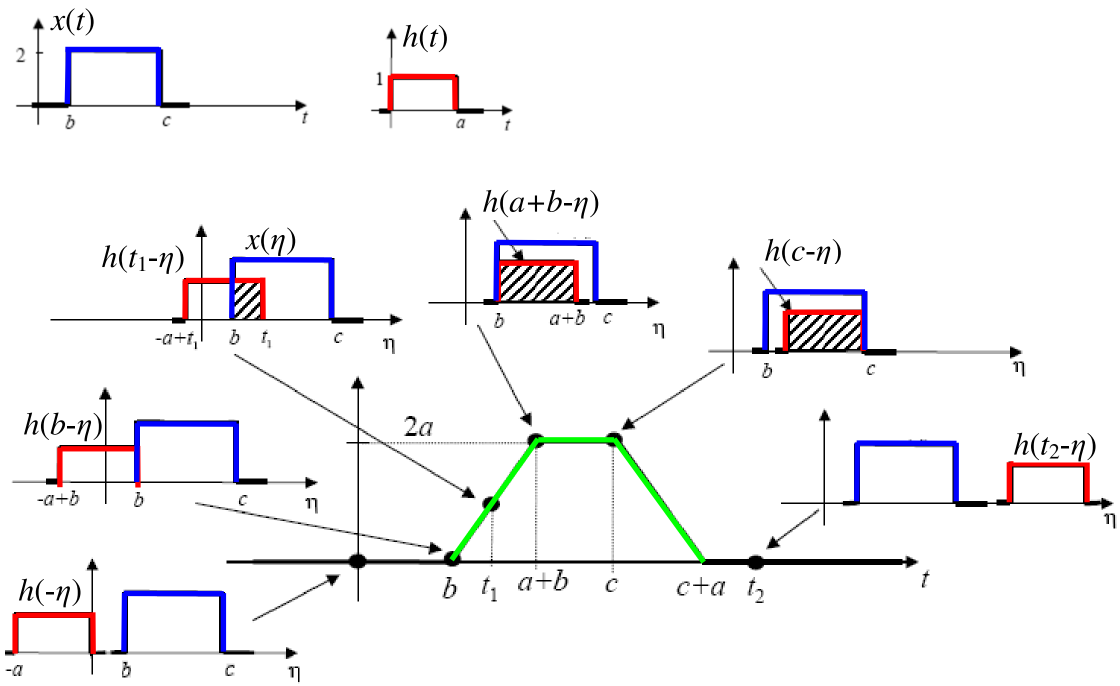
<u>Example 1</u>

Shown in the following simple example is the convolution of two identical windows, note that in this case $h(-\eta) = h(\eta)$. Here the amplitude is A=1, then the value of the convolution is exactly the intersection area between the two curves.



Example 2

In this example a slightly more complicated case is shown



The value of the convolution will be proportional at every point, to the intersection area of the two curves.